MATHEMATICS 201-105-RE
Linear Algebra
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MAPLE LAB

Part 1 – Matrices

Consider the following matrices.

$$A=\begin{bmatrix}1&3&5&7&9\\2&4&8&12&14\end{bmatrix},\quad B=\begin{bmatrix}1&\frac{1}{2}&\frac{1}{3}\\2&-3&4\\-1&0&2\end{bmatrix},\quad C=\begin{bmatrix}2&-1\\3&4\\1&7\end{bmatrix}\text{ and } D=\begin{bmatrix}a&2\\0&b\end{bmatrix}$$

1. Enter each of these matrices in Maple (giving them their respective names).
   a) Display the 2nd column of matrix B.
   b) Display the entry a_{23} from matrix A.
2. Evaluate the following.
   a) CA
   b) C^T
   c) CC^T
   d) $2B-3CC^T$
   e) B^4
   f) $B^{-1}C$
   g) $\operatorname{tr}\left(AA^T+C^TC\right)$
   h) $\operatorname{adj}(B)$
   i) $\det\left(CC^T\right)$
   j) $\frac{1}{\det(B)}\operatorname{adj}(B)-B^{-1}$
   k) B^{-6}
   l) CB
   m) CD
   n) D^{-2}

Part II – Systems of Linear Equations

1. Consider the following system of linear equations

$$\begin{aligned}2x-4y+8z&=-4\\5x+y-6z&=-12\\3x+7y-2z&=0\end{aligned}$$

   a) Solve the system using the "solve" command.
   b) Solve the system using the "linsolve" command.
   c) Solve the system using the inverse.
   d) Find the reduced row-echelon form of the augmented matrix corresponding to the system. Can you make out the solution from this?
   e) Find the *z* value using Cramer's Rule.

2. Repeat question 1 (when possible) with the following system.

$$\begin{aligned}2x+4y+5z-2w+t&=0\\x+2y+3z-w+2t&=-1\\6x+12y+10z+6t&=13\end{aligned}$$

   How do the different solutions compare?

Answers

Part 1

1. a) ½, -3, 0 b) 8

2. a) $\begin{bmatrix} 0 & 2 & 2 & 2 & 4 \\ 11 & 25 & 47 & 69 & 83 \\ 15 & 31 & 61 & 91 & 107 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 3 & 1 \\ -1 & 4 & 7 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 2 & -5 \\ 2 & 25 & 31 \\ -5 & 31 & 50 \end{bmatrix}$

d) $\begin{bmatrix} -13 & -5 & \frac{47}{3} \\ -2 & -81 & -85 \\ 13 & -93 & -146 \end{bmatrix}$ e) $\begin{bmatrix} \frac{16}{9} & \frac{-79}{6} & \frac{58}{3} \\ \frac{-250}{3} & \frac{329}{3} & \frac{-626}{9} \\ -12 & \frac{-23}{6} & \frac{55}{9} \end{bmatrix}$ f) $\begin{bmatrix} \frac{12}{11} & \frac{-23}{11} \\ \frac{37}{33} & \frac{6}{11} \\ \frac{23}{22} & \frac{27}{11} \end{bmatrix}$

g) 669 h) $\begin{bmatrix} -6 & -1 & 3 \\ -8 & \frac{7}{3} & \frac{-10}{3} \\ -3 & \frac{-1}{2} & -4 \end{bmatrix}$ i) 0

j) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ k) $\begin{bmatrix} \frac{-2152621}{143496441} & \frac{-172219}{860978646} & \frac{-5179991}{430489323} \\ \frac{1462018}{430489323} & \frac{641425}{1291467969} & \frac{-38789224}{1291467969} \\ \frac{2676763}{143496441} & \frac{625807}{430489323} & \frac{-13236538}{4304899323} \end{bmatrix}$

l) error m) $\begin{bmatrix} 2a & 4-b \\ 3a & 6+4b \\ a & 2+7b \end{bmatrix}$ n) $\begin{bmatrix} \frac{1}{a^2} & \frac{-2a-2b}{a^2b^2} \\ 0 & \frac{1}{b^2} \end{bmatrix}$

Part II

1. a) $x=-2, y=1, z=\frac{1}{2}$ b) $x=-2, y=1, z=\frac{1}{2}$ c) $x=-2, y=1, z=\frac{1}{2}$

d) $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$, solution $\left(-2,1,\frac{1}{2}\right)$ e) ½

2. a) $\left(\frac{11}{2}-2y+4t, y, -2-3t, \frac{1}{2}-3t, t\right)$

b) $\left(\frac{37}{6}-2t_1-\frac{4}{3}t_2, t_1, \frac{-5}{2}+t_2, t_2, \frac{1}{6}-\frac{1}{3}t_2\right)$

c) It can't be done that way. (A is not invertible).

d) $\begin{bmatrix} 1 & 2 & 0 & 0 & -4 & \frac{11}{2} \\ 0 & 0 & 1 & 0 & 3 & -2 \\ 0 & 0 & 0 & 1 & 3 & \frac{1}{2} \end{bmatrix}$, solution $\left(\frac{11}{2}-2s+4r, s, -2-3r, \frac{1}{2}-3r, r\right)$.

e) It can't be done, the determinant of A being undefined.

f) They do not all look alike, but each properly represents the solution set. (There is more than one way to express an infinite amount of solutions!)