MATHEMATICS 201-105-RE
Linear Algebra
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Assignment #4

This assignment is due ***Friday April 21*****.** Complete solutions are expected.

Question 1 (8 points)

Prove that the set $W=\{(t,3):t\in\mathbb{R}\}$ with the following definitions for vector sum and scalar multiplication is a vector space. (Verify all 10 axioms!)

$$(u_1,3)\oplus(v_1,3)=(u_1+v_1,3)$$
$$k\odot(u_1,3)=(ku_1,3)$$

Question 2 (4 points)

Does the set $S=\{x^2+x+1, x^2-x+3, 2x^2-x+5\}$ span P_2?

Question 3 (7 points)

Consider the set of symmetric 2×2 matrices $S_2=\left\{\begin{bmatrix} a & b \\ b & c\end{bmatrix}; a,b,c\in\mathbb{R}\right\}$.

a) Show that S_2 is a subspace of $M_{2,2}$, the set of 2×2 matrices.

b) Find a basis for S_2.

c) What is the dimension of S_2?

Question 4 (5 points)

For what values of t is the set $S=\{(2,-3,3),(3,-2,t),(-1,t,4)\}$ linearly independent?

Question 5 (5 points)

Do the following sets B form a basis for V?

a) $B=\left\{\begin{bmatrix} 1 & 2 \\ 3 & 4\end{bmatrix},\begin{bmatrix} 2 & 4 \\ 6 & 1\end{bmatrix},\begin{bmatrix} -1 & -2 \\ 3 & 5\end{bmatrix},\begin{bmatrix} 3 & 1 \\ -1 & 2\end{bmatrix},\begin{bmatrix} 2 & 2 \\ 2 & 2\end{bmatrix}\right\}$ $\quad V=M_{2,2}$

b) $B=\{(2,-1,3),(4,1,5),(-2,-5,-1)\}$ $\quad V=\mathbb{R}^3$

Question 6 (13 points)

Consider the subset $W = \{(t, 3t + 2s, s) \mid s, t \in \mathbb{R}\}$ of $\mathbb{R}^3$.

a) Show that W is a subspace of $\mathbb{R}^3$.
b) Find a basis for W.
c) Find the dimension of W.
d) Give a geometrical description of W (lines in symmetric form and planes in general form).
e) Find the coordinate vector for $\vec{v} = (4, 22, 5) \in W$ relative to the basis found in (b).

Question 7 (8 points)

Consider the set $A = \{(-1, 2, 3), (3, -2, 1), (11, -10, -3), (1, 0, 2)\}$

a) Find a basis for $\text{span}(A)$.
b) Find the dimension of $\text{span}(A)$.
c) Give a geometrical interpretation for $\text{span}(A)$ (lines in symmetric form and planes in general form).