MATHEMATICS 201-105-RE
Linear Algebra
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Assignment #3

This assignment is due ***Friday March 31.*** Complete solutions are expected. For questions 3 and 6, a print out of your Maple is expected, where each question (and answer) is clearly indicated.

Question 1 (10 points)

Consider the points $A(-3,-4)$ and $B(5,-2)$ and $C(-5,1)$.

a) Find the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
b) Find the orthogonal projection of $\overrightarrow{AB}$ onto $\overrightarrow{AC}$.
c) Find the area of the triangle *ABC*.
d) Find the equation of the line *L* (in parametric form and in general form) parallel to $L_1 : 3x-4y=12$ and passing through the point *C*.
e) Find the distance between the line $L_1 : 3x-4y=12$ and the point *B*.

Question 2 (10 points)

Consider the lines $L_1 : \dfrac{x+1}{2} = \dfrac{y-3}{4} = \dfrac{3-z}{5}$ and $L_2 : \begin{cases} x = 2+3t \\ y = 5-4t \\ z = 5 \end{cases}$.

a) Find the relationship between L_1 and L_2. (That is, are they parallel and distinct, parallel and equivalent, intersecting or skew?)
b) Find the equation of the line L_3 (in symmetric form) parallel to L_1 and passing through the point $P(-1,13,-16)$.
c) Find the distance between L_1 and $P(-1,13,-16)$.
d) Find the distance between L_1 and L_2.
e) Find the point *Q* on L_1 that is closest to the point $P(-1,13,-16)$.

Question 3 (6 points)

Consider the points $A(3,2,1)$, $B(-2,4,-6)$ and $C(4,0,-5)$. Using Maple,

a) find the equation of the line *l* (in vector form) passing through the points *A* and *B*;
b) plot the line *l* found in (a) along with the direction vector for the line;
c) find the distance between the point *C* and the line *l* found in (a).

Question 4 (8 points)

Consider the plane $\pi_1 : 3x - 2y + 4z = 6$.

a) Find the equation of the line *l* (in symmetric form) perpendicular to π_1 and passing through the point $P(-2, 5, -9)$.

b) Find the distance between the point $P(-2, 5, -9)$ and the plane π_1.

c) Find the point *Q* on π_1 that is closest to the point $P(-2, 5, -9)$.

d) Find the equation of the plane (in general form) perpendicular to π_1 and containing the line $l_1 : \frac{x-1}{2} = \frac{y}{2} = \frac{1-z}{3}$.

Question 5 (2 points)

Find the equation of the plane (in general form), if possible, containing the lines $L_1 : \frac{x+1}{2} = \frac{y-3}{4} = 2z$ and $L_2 : \frac{x+1}{4} = \frac{x-4}{8} = z - 3$.

Question 6 (10 points)

Consider the four points $A(-1, 3, 4)$, $B(3, 3, 0)$, $C(2, -1, 3)$ and $E(1, 3, 1)$. Using Maple,

a) find the equation of the plane π (in general form) passing through the points *A*, *B* and *C*;

b) find the equation of the line *l* (in vector form) passing through *E* and perpendicular to the plane π;

c) find the distance between the point *E* and the plane π;

d) plot the line *l*, the plane π and the normal vector;

e) find the volume of the tetrahedron *ABCE*.

Question 7 (4 points)

Consider the planes $\pi_1 : x + y - 2z = 5$ and $\pi_2 : 2x + y + 3z = 4$.

a) Find the intersection of the planes π_1 and π_2 (expressed in vector form).

b) Find the angle between the two planes.