MATHEMATICS 201-105-RE
Linear Algebra
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Assignment #1

This assignment is due ***Wednesday February 1, 2006***.

Question 1 (10 points)

Simplify (if possible).

a) $3\begin{bmatrix} 3 & -1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 & 3 \\ 5 & 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 3 & 4 \end{bmatrix}^T$

b) $tr\left(\begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}^2\right)$

Question 2 (10 points)

Solve the following system of linear equations using either Gaussian elimination or the Gauss-Jordan method.

$$\begin{aligned} 2x \qquad\quad + 18z + 7w &= 14 \\ x - 2y + 3z + w &= 4 \\ -3x + 8y - 3z + 2w &= -4 \end{aligned}$$

Question 3 (10 points)

Solve the following system equations.

$$\begin{aligned} \frac{1}{x} + \frac{2}{y} - \frac{4}{z} &= 1 \\ \frac{2}{x} + \frac{3}{y} + \frac{8}{z} &= 0 \\ -\frac{1}{x} + \frac{9}{y} + \frac{10}{z} &= 5 \end{aligned}$$

Question 4 (10 points)

Consider the following system of linear equations.

$$\begin{aligned} x + y + az &= a \\ x + y + a^2 z &= 2a \\ ax + a^2 y + z &= 3a \end{aligned}$$

Find all values of a such that

a) the system has no solutions
b) the system has a unique solution
c) the system has an infinite number of solutions.

Question 5 (10 points)

Consider the matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 1 & -2 \\ 4 & 1 & 2 \end{bmatrix}$. Find the inverse of A using Gauss-Jordan elimination.

Question 6 (10 points)

A matrix A is ***idempotent*** if $A^2 = A$.

a) Show that $A = \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$ is idempotent.
b) Prove that if A and is idempotent, then A^T is also idempotent.
c) Prove that if A is idempotent, then $(I - 2A)^{-1} = I - 2A$.