MATHEMATICS 201-103-RE
Differential Calculus
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X – Implicit Differentiation

1. Find $\frac{dy}{dx}$.

   a) $4x^2+9y^2=36$
   b) $xy^2+x^3+y^3=4$
   c) $x+3xy^2-x^2y=7y$
   d) $x^4y+\sqrt{xy}=2$
   e) $\frac{1}{x}+\frac{1}{y}=1$
   f) $\sqrt{x+y}+\sqrt{x-y}=6$
   g) $x^2=\frac{x+y}{x-y}$
   h) $\frac{x+y}{3x-y}=1$
   i) $\left(5x^2y+4\right)^7=x^3$
   j) $\sin^2 x+\cos^2 y=1$
   k) $\sin\left(x^2y^2\right)=x$
   l) $\cot\left(x+2y\right)=4xy$

2. Find an equation for the tangent line to the graph of the implicit function at the specified point.

   a) $y^3+yx+x^2-3y^2=0$ at $(0,3)$
   b) $x^{\frac{2}{3}}+y^{\frac{2}{3}}=4$ at $\left(-1,3\sqrt{3}\right)$
   c) $\left(x+y\right)^3=x^3+y^3$ at $(-1,1)$
   d) $y^2=\frac{x^3}{4-x}$ at $(2,2)$
   e) $x\cos y+y\sin x=\frac{\pi}{2}$ at $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$

3. Find $\frac{da}{dt}$ by implicit differentiation.

   a) $a^2t-2t=at^3$
   b) $t=\sin a$

4. The demand function for a certain commodity is given by $p=\frac{13}{1+x^2+x^3}$. Find the instantaneous rate of change of the number sold with respect to the price when $x=2$.

5. The demand function for a certain commodity is given by $p=\frac{500\,000}{2x^3+400x+5000}$ where p is the price in dollars and x is the demand in hundreds of units.

   a) Find the rate of change of demand with respect to price for a demand of 10 000 units.
   b) Find the rate of change of price with respect to demand for a demand of 10 000 units.

ANSWERS

1. a) $\frac{dy}{dx}=\frac{-4x}{9y}$

   b) $\frac{dy}{dx}=-\frac{y^2+3x^2}{2xy+3y^2}$

   c) $\frac{dy}{dx}=\frac{2xy-3y^2-1}{6xy-x^2-7}$

   d) $\frac{dy}{dx}=-\frac{8x^3y\sqrt{xy}+y}{2x^4\sqrt{xy}+x}$

   e) $\frac{dy}{dx}=\frac{-y^2}{x^2}$

   f) $\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$

   g) $\frac{dy}{dx}=\frac{x^3-2x^2y+xy^2+y}{x}$

   h) $\frac{dy}{dx}=\frac{y}{x}$

   i) $\frac{dy}{dx}=\frac{3x-70y\left(5x^2y+4\right)^6}{35x\left(5x^2y+4\right)^6}$

   j) $\frac{dy}{dx}=\frac{\sin x\cos x}{\sin y\cos y}$

   k) $\frac{dy}{dx}=\frac{1-2xy^2\cos\left(x^2y^2\right)}{2x^2y\cos\left(x^2y^2\right)}$

   l) $\frac{dy}{dx}=-\frac{4y+\csc^2\left(x+2y\right)}{4x+2\csc^2\left(x+2y\right)}$

2. a) $y=\frac{-1}{3}x+3$ b) $y=\sqrt{3}x+4\sqrt{3}$ c) $y=-x$

   d) $y=2x-2$ e) $y=\frac{\pi}{2}$

3. a) $\frac{da}{dt}=\frac{3at^2-a^2+2}{2at-t^3}$ b) $\frac{da}{dt}=\sec a$

4. $\left.\frac{dx}{dp}\right|_{x=2}=\frac{-13}{16}$

5. a) $\left.\frac{dx}{dp}\right|_{x=100}\approx -138$ Thus when the demand is 10 000 units, the demand is decreasing at the rate of 13 800 units per dollar increase in price.

   b) $\left.\frac{dp}{dx}\right|_{x=100}\approx -0.0072$ Thus when the demand is 10 000 units, the price is decreasing at the rate of 0.0072¢ per increase of one unit.