CEGEP CHAMPLAIN ST. LAWRENCE CHAMPLAIN REGIONAL COLLEGE

MATHEMATICS 201-103-RE
Differential Calculus
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VIII – The Chain Rule

1. Differentiate the function.

a) $f(x)=(3x^2-4x)^{15}$

b) $f(x)=\left(\sqrt{x}+\frac{1}{x}\right)^{12}$

c) $f(x)=\dfrac{1}{(2x-5)^7}$

d) $f(t)=\sqrt{t^4-t+3}$

e) $f(x)=\sqrt[5]{x^3-2x+1}$

f) $f(x)=\tan\sqrt{x}$

g) $f(x)=2\cos 4x$

h) $f(x)=3\cos^5 x$

i) $f(x)=3\cos^5(10x)$

j) $f(x)=2\sec^2(x^6)$

k) $f(x)=\sin(\cos x)$

l) $f(x)=\dfrac{\sin 3x}{1+\cos 3x}$

m) $f(z)=\csc^2(12z)-\cot^2(12z)$

n) $f(x)=\cos^3\left(\dfrac{x}{x+1}\right)$

o) $f(x)=(\sec x-1)^{-4}$

p) $f(x)=\sqrt{x+\cos^3(5x)}$

q) $f(x)=(3x^2-1)^4(2x+4)^3$

r) $f(x)=(2x-5)^2(x^2+4)^3$

s) $f(x)=\dfrac{(2x+3)^3}{(4x^2-1)^8}$

t) $f(x)=\left(\dfrac{x^2+x}{1-2x}\right)^4$

u) $f(x)=\sin^3(x^2)\cos^4(x^2)$

v) $f(x)=\sqrt{\csc^3(5x)}\cot^4(5x)$

w) $f(x)=(2x-1)^{\frac{3}{2}}(5x+2)^{\frac{1}{3}}$

x) $f(t)=\sqrt{t}\sec\sqrt{t}-\frac{1}{t}\tan\frac{1}{t}$

y) $f(x)=\dfrac{\sqrt{3x-1}}{\sqrt[3]{2x+5}}$

z) $f(z)=\dfrac{1}{\sqrt{z^2+1}}-\sqrt{1+\sin^2 z}+\sqrt[5]{\pi}$

2. Find $\dfrac{dy}{dx}$.

a) $y=x^2\sin^3(2x)$

b) $y=\tan^4\left(\sqrt{3x-1}\right)$

c) $y=\sin(\cos(\tan 5x))$

d) $y=\sec^2 x\tan^3 x$

e) $y=\dfrac{(4x^2-1)^5}{(1-2x)^3}$

f) $y=\left(\dfrac{16x+1}{2x-1}\right)^{-4}$

g) $y = \dfrac{x^2+1}{x(x+2)^3}$

h) $y = \sin^4(3x^2)\cos^3(3x^2)$

i) $y = (4x+3)^4 \sqrt[3]{2x+1}$

3. Find an equation for the tangent line to the graph at the specified point.
   a) $y = \sec^3\left(\frac{\pi}{2} - x\right)$ $\quad x = \frac{\pi}{2}$
   b) $y = \left(x - \frac{2}{x}\right)^4$ $\quad x = 2$

4. Find all points on the graph of the function at which the tangent line is horizontal.
   a) $y = (3x-5)^4(2x+1)^5$
   b) $y = \dfrac{(x^2-9)^4}{(x^2+9)^5}$

5. Find the equation of all tangent lines to the graph of $f(x) = (3x-1)^4$ that are parallel to $y = 12x + 5$.

6. Find the equation of all tangent lines to the graph of $f(x) = \dfrac{-9}{(3x+2)^3}$ that are parallel to $y = 16x - 2$.

ANSWERS

1. a) $f'(x)=30(3x-2)(3x^2-4x)^{14}$

b) $f'(x)=12\left(\frac{1}{2\sqrt{x}}-\frac{1}{x^2}\right)\left(\sqrt{x}+\frac{1}{x}\right)^{11}$

c) $f'(x)=\dfrac{-14}{(2x-5)^8}$

d) $f'(t)=\dfrac{4t^3-1}{2\sqrt{t^4-t+3}}$

e) $f'(x)=\dfrac{3x^2-2}{5\left(x^3-2x+1\right)^{\frac{4}{5}}}$

f) $f'(x)=\dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}$

g) $f'(x)=-8\sin 4x$

h) $f'(x)=-15\cos^4 x\sin x$

i) $f'(x)=-150\cos^4(10x)\sin(10x)$

j) $f'(x)=24x^5\sec^2\left(x^6\right)\tan\left(x^6\right)$

k) $f'(x)=-\sin x\cos(\cos x)$

l) $f'(x)=\dfrac{3\cos 3x+3}{(1+\cos 3x)^2}=\dfrac{3}{1+\cos 3x}$

m) $f'(z)=0$

n) $f'(x)=\dfrac{-3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)}{(x+1)^2}$

o) $f'(x)=\dfrac{-4\sec x\tan x}{(\sec x-1)^5}$

p) $f'(x)=\dfrac{1-15\cos^2(5x)\sin(5x)}{2\sqrt{x+\cos^3(5x)}}$

q) $f'(x)=6\left(11x^2+16x-1\right)(2x+4)^2\left(3x^2-1\right)^3$

r) $f'(x)=2\left(8x^2-15x+8\right)(2x-5)\left(x^2+4\right)^2$

s) $f'(x)=\dfrac{-2(2x+3)^2\left(52x^2+96x+3\right)}{\left(4x^2-1\right)^9}$

t) $f'(x)=\dfrac{-4\left(x^2+x\right)^3\left(2x^2-2x-1\right)}{(1-2x)^5}$

u) $f'(x)=6x\sin^2\left(x^2\right)\cos^5\left(x^2\right)-8x\sin^4\left(x^2\right)\cos^3\left(x^2\right)$

v) $f'(x)=\frac{-15}{2}\csc^{\frac{3}{2}}(5x)\cot^5(5x)-20\csc^{\frac{7}{2}}(5x)\cot^3(5x)$

w) $f'(x)=\dfrac{(55x+13)\sqrt{2x-1}}{3(5x+2)^{\frac{2}{3}}}$

x) $f'(t)=\dfrac{\sec\sqrt{t}}{2\sqrt{t}}+\frac{1}{2}\sec\sqrt{t}\tan\sqrt{t}+\dfrac{\sec^2\frac{1}{t}+t\tan\frac{1}{t}}{t^3}$

y) $f'(x)=\dfrac{6x+49}{6(3x-1)^{\frac{1}{2}}(2x+5)^{\frac{4}{3}}}$

z) $f'(z)=\dfrac{-z}{\left(z^2+1\right)^{\frac{3}{2}}}-\dfrac{\sin z\cos z}{\sqrt{1+\sin^2 z}}$

2. a) $\frac{dy}{dx} = 2x\sin^3(2x) + 6x^2\sin^2(2x)\cos(2x)$

   b) $\frac{dy}{dx} = \frac{6\tan^3\sqrt{3x-1}\sec^2\sqrt{3x-1}}{\sqrt{3x-1}}$

   c) $\frac{dy}{dx} = -5\cos(\cos(\tan 5x))\sin(\tan(5x))\sec^2(5x)$

   d) $\frac{dy}{dx} = 2\sec^2 x\tan^4 x + 3\sec^4 x\tan^2 x$

   e) $\frac{dy}{dx} = -2(2x+1)^4(2x-1)(14x-3)$

   f) $\frac{dy}{dx} = \frac{72(2x-1)^3}{(16x+1)^5}$

   g) $\frac{dy}{dx} = \frac{2x^3 - 2x^2 + 4x + 2}{x^2(x+2)^4}$

   h) $\frac{dy}{dx} = 24x\sin^3(3x^2)\cos^4(3x^2) - 18x\sin^5(3x^2)\cos^2(3x^2)$

   i) $\frac{dy}{dx} = \frac{2(52x+27)(4x+3)^3}{3(2x+1)^{\frac{2}{3}}}$

3. a) $y = 1$ b) $y = 6x - 11$

4. a) $x = \frac{19}{27}, \frac{5}{3}, \frac{-1}{2}$ b) $x = 0, \pm 3, \pm 9$

5. $y = 12x - 7$

6. $y = 16x$ and $y = 16x + \frac{64}{3}$