MATHEMATICS 201-103-RE
Differential Calculus
Martin Huard
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VII –Derivatives of Trigonometric Functions

1. Convert from degrees to radians.
   a) $135°$ b) $900°$ c) $-315°$

2. Convert from radians to degrees.
   a) $\frac{8\pi}{3}$ b) $\frac{-7\pi}{12}$ c) 3

3. Evaluate exactly (without the use of a calculator).
   a) $\sin\left(\frac{2\pi}{3}\right)$ b) $\tan\left(\frac{-\pi}{4}\right)$ c) $\cos\left(\frac{5\pi}{6}\right)$ d) $\sin\left(\frac{5\pi}{2}\right)$
   e) $\sec\left(\frac{7\pi}{4}\right)$ f) $\cot\pi$ g) $\csc\left(\frac{7\pi}{6}\right)$ h) $\cot\left(\frac{3\pi}{4}\right)$

4. Use identities to evaluate exactly.
   a) $\sin(75°)$ b) $\cos\left(\frac{\pi}{12}\right)$ c) $\cos(165°)$

5. Find the limit.
   a) $\lim\limits_{x\to 0}\frac{\sin 4x}{2x}$ b) $\lim\limits_{x\to 0}\frac{\sin 3x}{\sin 7x}$ c) $\lim\limits_{\theta\to 0}\frac{\sin(\cos\theta)}{\cos\theta}$
   d) $\lim\limits_{x\to 0}\frac{\sin x-\cos x\sin x}{x^2}$ e) $\lim\limits_{t\to 0}\frac{\tan 2t}{3t}$ f) $\lim\limits_{x\to 0}\frac{\cot x}{\csc 2x}$
   g) $\lim\limits_{x\to 0}(\sin 3x\csc 12x)$ h) $\lim\limits_{x\to 0}\frac{2x+\sin x}{x}$ i) $\lim\limits_{x\to 1}\frac{\sin(x-1)}{2x-2}$

6. Differentiate the function.
   a) $f(x)=2\cos x-5\sin x$ b) $f(x)=\frac{\sin x}{x}$ c) $f(x)=\sec x-5\tan x$
   d) $f(x)=\sec x\tan x$ e) $f(t)=t^3\csc t-t\cot t$ f) $f(x)=\frac{\cot x}{1+\cot x}$
   g) $f(x)=\frac{\csc x}{\tan x}$ h) $f(x)=\frac{x^2+1}{\cos x-1}$ i) $f(x)=\frac{1-\sin x}{1+2\sin x}$
   j) $f(x)=\sqrt{x}\sin x+\frac{\cos x}{\sqrt{x}}$ k) $f(\theta)=\frac{3\cos\theta}{2\cos\theta-\sin\theta}$ l) $f(x)=\frac{\tan x}{1+x\tan x}$

7. Find the equation for the tangent and normal lines to the graph of each function at the given point.
   a) $f(x) = 2\sin x$ at $\left(\frac{\pi}{6}, 1\right)$.
   b) $f(x) = 3\tan x$ at $\left(\frac{3\pi}{4}, -3\right)$.
   c) $f(x) = x + \cos x$ at $(\pi, \pi - 1)$.
   d) $f(x) = \sec x + \csc x$ at $\left(\frac{\pi}{4}, 2\sqrt{2}\right)$.
   e) $f(x) = 2\cot x$ at $\left(\frac{\pi}{6}, 2\sqrt{3}\right)$.

8. For what values of x does the graph of $f(x) = x + 2\cos x$ have a horizontal tangent?

ANSWERS

1. a) $\frac{3\pi}{4}$ b) 5π c) $\frac{-7\pi}{4}$
2. a) $480°$ b) $-105°$ c) $\frac{540}{\pi}°$
3. a) $\frac{\sqrt{3}}{2}$ b) -1 c) $\frac{-\sqrt{3}}{2}$ d) 1 e) $\sqrt{2}$ f) $\nexists$ g) -2 h) -1
4. a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ b) $\frac{\sqrt{6}+\sqrt{2}}{4}$ c) $\frac{-\sqrt{6}-\sqrt{2}}{4}$
5. a) 2 b) $\frac{3}{7}$ c) $\sin 1$ d) 0 e) $\frac{2}{3}$ f) 2 g) $\frac{1}{4}$ h) 3 i) $\frac{1}{2}$
6. a) $f'(x) = -2\sin x - 5\cos x$ b) $f'(x) = \frac{x\cos x - \sin x}{x^2}$
   c) $f'(x) = \sec x \tan x - 5\sec^2 x$ d) $f'(x) = \sec x \tan^2 x + \sec^3 x$
   e) $f'(t) = 3t^2 \csc t - t^3 \csc t \cot t - \cot t + t\csc^2 t$
   f) $f'(x) = \frac{-\csc^2 x}{(1+\cot x)^2}$
   g) $f'(x) = \frac{-\csc x - \csc x \sec^2 x}{\tan^2 x}$ h) $f'(x) = \frac{2x\cos x - 2x + x^2 \sin x + \sin x}{(\cos x - 1)^2}$
   i) $f'(x) = \frac{-3\cos x}{(1+2\sin x)^2}$ j) $f'(x) = -\frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x - \frac{\cos x}{2x^{\frac{3}{2}}}$
   k) $f'(\theta) = \frac{3}{(2\cos\theta - \sin\theta)^2}$ l) $f'(x) = \frac{1}{(1+x\tan x)^2}$
7. a) $y_T = \sqrt{3}x - \frac{\sqrt{3}\pi}{6} + 1$, $y_N = -\frac{\sqrt{3}}{3}x + \frac{\sqrt{3}\pi}{18} + 1$
   b) $y_T = 6x - \frac{9\pi}{2} - 3$, $y_N = \frac{-1}{6}x + \frac{\pi}{8} - 3$
   c) $y_T = x - 1$, $y_N = -x + 2\pi - 1$
   d) $y_T = 2\sqrt{2}$, $x_N = \frac{\pi}{4}$
   e) $y_T = -8x + \frac{4\pi}{3} + 2\sqrt{3}$, $y_N = \frac{1}{8}x - \frac{\pi}{48} + 2\sqrt{3}$
8. $x = \frac{\pi}{6} \pm 2\pi n,\ \frac{5\pi}{6} \pm 2\pi n \quad n \in \mathbb{Z}$