

VI – Rules of Differentiation

1. Differentiate each function.

a) $f(x) = 4x^3$

c) $f(x) = \frac{3x^5}{10} - \sqrt{2}$

e) $f(x) = x^{-3} + \frac{1}{x}$

g) $f(t) = \frac{1}{3t^2} - \frac{1}{2t} + 3$

i) $f(t) = \frac{1}{3}t^{-3} - 3t^3 + \frac{1}{\sqrt{t}}$

k) $f(t) = \sqrt[3]{t^2} - \frac{1}{\sqrt{t^5}} + \sqrt[5]{t} + \sqrt{3}$

m) $f(x) = (x^3 - x)(x^2 + 2x)$

o) $f(x) = (x^2 + 1)(x^2 + 2)$

q) $f(x) = \sqrt{x}(\sqrt{x} + 1)$

s) $f(x) = \frac{x^2 + 1}{x^2 + 4}$

u) $f(x) = \frac{-3}{x^2 + 2x - 5}$

w) $f(x) = \frac{x^2 + 3x + 5}{x}$

y) $f(x) = \frac{ax + b}{cx + d}$

b) $f(x) = \frac{3}{2}x^6 - 3x + 1$

d) $f(x) = \frac{x^{10}}{2} - \frac{x^5}{5} + \frac{1}{4}$

f) $f(x) = x^5 + x^{-5} + \frac{1}{x^3}$

h) $f(x) = \sqrt{x} + \frac{3}{2x} - 5x^3$

j) $f(x) = x^{\frac{2}{3}} - \frac{4}{x^{\frac{5}{7}}} + 3x^{\frac{1}{2}} - 4x^{\sqrt{2}}$

l) $f(x) = x^2(2x^3 - 1)$

n) $f(x) = x^4(x^2 + x + 1)$

p) $f(x) = \frac{2x + 3}{4x - 5}$

r) $f(x) = \frac{x^3 - x^2 + 1}{x}$

t) $f(t) = \frac{t^3}{3t^4 + 1}$

v) $f(x) = \frac{2x^2 + x + 1}{x^2 - 3x + 1}$

x) $f(x) = \frac{\sqrt{x} + 1}{\sqrt{x} - 1}$

2. Find $f'(3)$.

a) $f(x) = \frac{1}{4}x^4$

c) $f(x) = (x^3 + 1)(x - 4)$

b) $f(x) = \frac{1}{x} + 2\sqrt{x} - 3$

d) $f(x) = \frac{x^3 - 2}{x^2 + x}$

ANSWERS

1. a) $f'(x) = 12x^2$ b) $f'(x) = 9x^5 - 3$ c) $f'(x) = \frac{3}{2}x^4$
 d) $f'(x) = 5x^9 - x^4$ e) $f'(x) = \frac{-3}{x^4} - \frac{1}{x^2}$ f) $f'(x) = 5x^4 - \frac{5}{x^6} - \frac{3}{x^4}$
 g) $f'(t) = \frac{-2}{3t^3} + \frac{1}{2t^2}$ h) $f'(x) = \frac{1}{2\sqrt{x}} - \frac{3}{2x^2} - 15x^2$ i) $f'(t) = \frac{-1}{t^4} - 9t^2 - \frac{1}{2t^{\frac{3}{2}}}$
 j) $f'(x) = \frac{2}{3x^{\frac{1}{3}}} + \frac{20}{7x^{\frac{12}{7}}} + \frac{3}{2\sqrt{x}} - 4\sqrt{2}x^{\sqrt{2}-1}$ k) $f'(t) = \frac{2}{3t^{\frac{1}{3}}} + \frac{5}{2t^{\frac{7}{2}}} + \frac{1}{5t^{\frac{4}{5}}}$
 l) $f'(x) = 10x^4 - 2x$ m) $f'(x) = 5x^4 + 8x^3 - 3x^2 - 4x$
 n) $f'(x) = 6x^5 + 5x^4 + 4x^3$ o) $f'(x) = 4x^3 + 6x$ p) $f'(x) = \frac{-22}{(4x-5)^2}$
 q) $f'(x) = 1 + \frac{1}{2\sqrt{x}}$ r) $f'(x) = \frac{2x^3 - x^2 - 1}{x^2}$ s) $f'(x) = \frac{6x}{(x^2 + 4)^2}$
 t) $f'(t) = \frac{-3t^6 + 3t^2}{(3t^4 + 1)^2}$ u) $f'(x) = \frac{6x + 6}{(x^2 + 2x - 5)^2}$ v) $f'(x) = \frac{4 + 2x - 7x^2}{(x^2 - 3x + 1)^2}$
 w) $f'(x) = \frac{x^2 - 5}{x^2}$ x) $f'(x) = \frac{-1}{\sqrt{x}(\sqrt{x} - 1)^2}$ y) $f'(x) = \frac{ad - cb}{(cx + d)^2}$
2. a) 27 b) $\frac{\sqrt{3}}{3} - \frac{1}{9}$ c) 1 d) $\frac{149}{144}$
3. a) 8 b) -9 c) 22 d) $\frac{1}{8}$
4. a) $y_T = 6x - 3$ b) $y_T = \frac{7}{8}x + \frac{3}{2}$ c) $y_T = -2x + 7$ d) $y_T = 8$
 $y_N = \frac{-1}{6}x + \frac{28}{3}$ $y_N = \frac{-8}{7}x + \frac{67}{7}$ $y_N = \frac{1}{2}x + 2$ $x_N = 4$
5. a) (1, 8) (3, 4) b) $(-2, \frac{-1}{4})$ c) (1, 2) d) (0, 0) (-2, -4)
6. a) -1 b) 1
7. $(\frac{9}{2}, \frac{101}{4})$ $y = 9x - \frac{61}{4}$
8. a) $y = 3x - \frac{13}{3}$ b) $y = -x + 1$ c) $y = \frac{1}{2}x + \frac{7}{2}$, $y = \frac{1}{2}x - \frac{1}{2}$ d) $y = \frac{1}{4}x + 1$
 e) $y = -2x$, $y = -2x + 3\sqrt{3}$, $y = -2x - 3\sqrt{3}$ f) $y = -5x - \frac{19}{3}$, $y = -5x + \frac{13}{3}$
9. $a = \frac{-1}{2}$, $b = 2$