MATHEMATICS 201-103-RE
Differential Calculus
Martin Huard
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V – Tangents and Derivatives

1. Find the slope of the tangent line (using the definition with *t*) for the curve $y = f(x)$ at the given point.
   a) $y = 5x - 3$ at $x = -2$
   b) $y = 3x^2 - 1$ at $x = 1$
   c) $y = 2x^2 - 4x + 5$ at $x = -1$
   d) $y = \sqrt{x+1}$ at $x = 8$
   e) $y = \dfrac{1}{2x-3}$ at $x = 2$

2. Find the equation of the tangent line for each of the curves in question 1 to the curve $y = f(x)$ at the given point.

3. Find the slope of the tangent line (using the definition with Δx) for the curve $y = f(x)$ at the given point and use your result to find the equation of the tangent line.
   a) $y = 3x^2 - 2x + 1$ at $x = 3$
   b) $y = x - 5x^2$ at $x = 1$
   c) $y = x^3 + 2x + 1$ at $x = -1$
   d) $y = \sqrt{2-x} + 3$ at $x = -2$
   e) $y = \dfrac{x}{x+1}$ at $x = 3$

4. Find the derivative of the function by direct use of the definition.
   a) $f(x) = 4x - 3$
   b) $f(x) = 3x^2$
   c) $f(x) = 2 - x^2$
   d) $f(x) = 2x^2 - 3x + 4$
   e) $f(x) = 5 - x - 3x^2$
   f) $f(x) = 2x^3 - 1$
   g) $f(x) = \sqrt{x} + 1$
   h) $f(x) = \sqrt{x+1}$
   i) $f(x) = \sqrt{5x+1}$
   j) $f(x) = \sqrt{x^2+1}$
   k) $f(x) = \dfrac{2}{3+x}$
   l) $f(x) = \dfrac{x+1}{x+4}$
   m) $f(x) = \dfrac{1}{\sqrt{x+1}}$
   n) $f(x) = \dfrac{1}{x^2+1}$

5. Find the derivative of the function by direct use of the alternate definition.

a) $f(x)=3+4x^2$ b) $f(x)=5x-x^2$

c) $f(x)=x^3-8$ d) $f(x)=\sqrt{x-3}$

e) $f(x)=\dfrac{x}{x-3}$ f) $f(x)=\dfrac{1}{x^2}$

6. For each of the functions $f(x)$,

i) find $f'(x)$

ii) the equation of the tangent line at $x=a$ using your answer for (i).

a) $f(x)=2x^2-3$ at $x=3$

b) $f(x)=x^3+x^2$ at $x=-1$

c) $f(x)=\sqrt{1-x}$ at $x=-3$

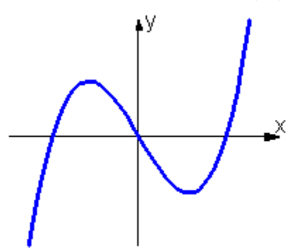
d) $f(x)=\dfrac{1}{x}+x$ at $x=1$

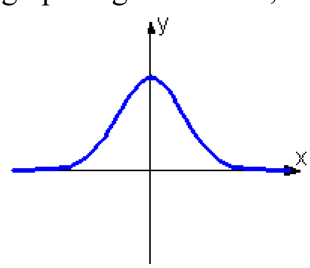
7. At what point does the curve $y=f(x)$ have a horizontal tangent?

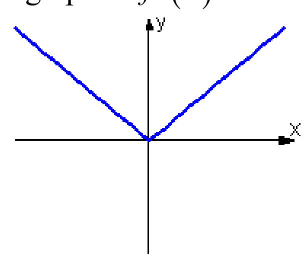
a) $f(x)=x^2-x$ b) $f(x)=3x^3-x$

c) $f(x)=x^4$

8. For each of the function $f(x)$ whose graph is given below, sketch the graph of $f'(x)$.

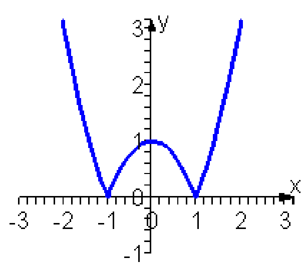
a) 

b) 

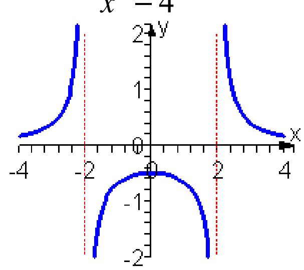
c) 

9. Determine the x values for which the function is not differentiable.

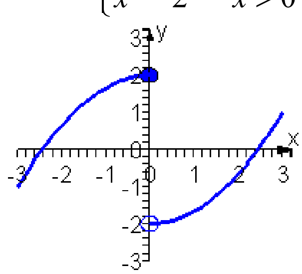
a) $f(x)=\left|x^2-1\right|$



b) $f(x)=\dfrac{2}{x^2-4}$



c) $f(x)=\begin{cases}2-x^2 & x\le 0\\ x^2-2 & x>0\end{cases}$



10. Show that the function $f(x)=|x-4|$ is not differentiable at $x=4$.

ANSWERS

1. a) 5 b) 6 c) -8 d) $\frac{1}{6}$ e) -2
2. a) $y=5x-3$ b) $y=6x-4$ c) $y=-8x+3$ d) $y=\frac{1}{6}x+\frac{5}{3}$ e) $y=-2x+5$
3. a) $y=16x-26$ b) $y=-9x+5$ c) $y=5x+3$ d) $y=\frac{-1}{4}x+\frac{9}{2}$ e) $y=\frac{1}{16}x+\frac{9}{16}$
4. a) $f'(x)=4$ b) $f'(x)=6x$ c) $f'(x)=-2x$ d) $f'(x)=4x-3$

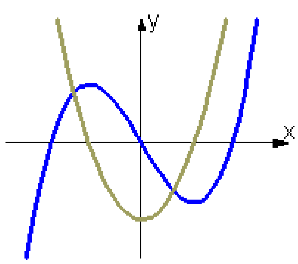
   e) $f'(x)=-1-6x$ f) $f'(x)=6x^2$ g) $f'(x)=\dfrac{1}{2\sqrt{x}}$ h) $f'(x)=\dfrac{1}{2\sqrt{x+1}}$

   i) $f'(x)=\dfrac{5}{2\sqrt{5x+1}}$ j) $f'(x)=\dfrac{x}{\sqrt{x^2+1}}$ k) $f'(x)=\dfrac{-2}{(3+x)^2}$ l) $f'(x)=\dfrac{3}{(x+4)^2}$

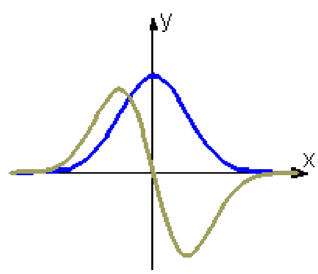
   m) $f'(x)=\dfrac{-1}{2(x+1)^{\frac{3}{2}}}$ n) $f'(x)=\dfrac{-2x}{(x^2+1)^2}$
5. a) $f'(x)=8x$ b) $f'(x)=5-2x$ c) $f'(x)=3x^2$ d) $f'(x)=\dfrac{1}{2\sqrt{x-3}}$

   e) $f'(x)=\dfrac{-3}{(x-3)^2}$ f) $f'(x)=\dfrac{-2}{x^3}$
6. a) $f'(x)=4x$ $y=12x-21$ b) $f'(x)=3x^2+2x$ $y=x+1$

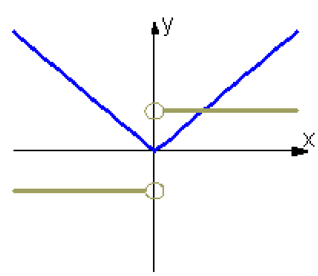
   c) $f'(x)=\dfrac{-1}{2\sqrt{1-x}}$ $y=\frac{-1}{4}x+\frac{5}{4}$ d) $f'(x)=\dfrac{-1}{x^2}+1$ $y=2$
7. a) $\left(\frac{1}{2},\frac{-1}{4}\right)$ b) $\left(\frac{1}{3},\frac{-2}{9}\right)$ $\left(\frac{-1}{3},\frac{2}{9}\right)$ c) $(0,0)$
8. a)



   b)



   c)



9. a) $x=\pm1$ b) $x=\pm2$ c) $x=0$
10. $f'_{-}(4)=\lim\limits_{\Delta x\to0^-}\dfrac{f(4+\Delta x)-f(4)}{\Delta x}=-1$ $\therefore f'(4)\nexists$

    $f'_{+}(4)=\lim\limits_{\Delta x\to0^+}\dfrac{f(4+\Delta x)-f(4)}{\Delta x}=1$