MATHEMATICS 201-103-RE
Differential Calculus
Martin Huard
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IX – Rates of Change

1. Suppose that the proportion P of voters who recognize a candidate's name t months after the start of the campaign is given by
$$P(t)=\frac{12t}{t^2+100}+0.25$$
   a) Find the rate of change of P when $t=6$.
   b) One month prior to the election, is it better for $P'(t)$ to be positive or negative?

2. The daily sales S (in thousands of dollars) attributed to an advertising campaign are given by
$$S=1+\frac{3}{t+3}-\frac{18}{(t+3)^2}$$
where t is the number of weeks the campaign runs.
   a) What is the rate of change of sales at $t=8$ and at $t=10$?
   b) Should the campaign be continued after the 10th week? Explain.

3. Wildlife biologists predict that the population N of a certain endangered species after t years will be given by the equation $N=(3t+150)(50-t)$ for $0\le t\le 50$ years. If this prediction is correct, find the rate of change 20 years from now. What does your answer represent?

4. The strength of a person's reaction to a certain drug is given by $R(Q)=Q\sqrt{C-\frac{1}{3}Q}$ where Q represents the quantity of the drug given to the patient and C is a constant.
   a) The derivative $R'(Q)$ is called the *sensitivity* to the drug. Find $R'(Q)$.
   b) Find the sensitivity to the drug if $C=59$ and a patient is given 87 units of the drug.

5. The revenue received from the sale of electric fans is seasonal, with maximum revenue in the summer. Let the revenue received from the sale of fans be approximated by
$$R(t)=100\cos\left(\tfrac{\pi}{6}t\right)+120$$
where t is time in months, measured from July 1.
   a) Find the rate of change of the revenue for August 1.
   b) Find the rate of change of the revenue for January 1.
   c) Find the rate of change of the revenue for June 1.

6. The displacement (in meters) of a particle moving in a straight line is given by $s=3t^2+t-1$, where t is measured in seconds.
   a) Find the average velocity over the time interval $[3,4]$.
   b) Find the instantaneous velocity when $t=3$.

7. If a ball is thrown into the air with a velocity of 10 m/s, its height (in meters) after t seconds is given by $y = 10t - 4.9t^2$.
   a) Find the velocity after 2 seconds.
   b) When will the ball reach its highest point?
   c) When will the ball be back on the ground?

8. The time T (in minutes) it takes to memorize a list on n items is given by $T(n) = 3n\sqrt{n-3}$. Find the rate of change of T when $n = 12$.

9. A division of Dif Industries manufactures the Futura model microwave oven. The daily cost (in dollars) of producing these microwave ovens is
   $$C(x) = 0.0002x^3 - 0.06x^2 + 120x + 5000$$
   where x stands for the number of units produced.
   a) Find the additional cost when the production increases from 100 to 101 ovens.
   b) Find the marginal cost when $x = 100$.

10. The cost (in dollars) for producing x stereo systems is given by $C(x) = \frac{1}{5}x^2 + 6x + 50$ while the demand is given by $p(x) = \frac{1000}{x^2} + 1000$. Find the marginal cost, revenue and profit for a production level of 10 stereos.

11. An analyst has found that a company's costs and revenues in dollars for one product are given by
    $$C(x) = x\sqrt{x^2 + 100} + 300 \qquad R(x) = 120x - x^2$$
    respectively, where x is the number of items produced. Find the marginal cost, revenue and profit for a production level of 24 units.

12. A manufacturer has been selling 1000 televisions sets a week at $450 each. A market survey indicates that for each $10 rebate offered to the buyer, the number of sets sold will increase by 100 per week.
    a) Find the demand function.
    b) Find the marginal revenue.
    c) If the cost function is $C(x) = 68000 + 150x$, find the marginal profit when selling 1200 televisions.

13. When the admission price to a hockey game was $25, 24 000 tickets were sold. When the price was raised to $30, only 20 000 tickets were sold. Assume that the demand function is linear and that the variable and fixed costs for the arena are $1.00 and $75 000 respectively.
    a) Find the profit P as a function of x, the number of tickets sold.
    b) When is the marginal profit equal to zero?
    c) Find the profit when the marginal profit is equal to zero.

ANSWERS

1. a) $\frac{12}{289} \approx 4.15\%$ per month

   b) It is better for $P'(t)$ to be positive, that is to have increasing recognition.
2. a) 2.25 \$/week and -1.37 \$/week b) No since the daily sales are decreasing.
3. –120 animals per year. The population is decreasing at a rate of 120 animals per year.
4. a) $R'(Q) = \dfrac{6C - 3Q}{2\sqrt{9C - 3Q}}$ b) $\dfrac{31\sqrt{30}}{60}$
5. a) $\frac{-25\pi}{3} \approx -26.2$ \$/month b) 0 \$/year c) $\frac{25\pi}{3} \approx 26.2$ \$/month
6. a) 22 m/s b) 19 m/s
7. a) -9.6 m/s b) 1.02 seconds c) 2.04 seconds
8. 15 minutes/item
9. a) \$114.0002 b) \$114.0000
10. Marginal cost: \$10 Marginal revenue: \$990 Marginal profit: \$980
11. Marginal cost: \$48.15 Marginal revenue: \$72.00 Marginal profit: \$23.85
12. a) $p(x) = 550 - \frac{1}{10}x$ b) $R'(x) = 550 - \frac{1}{5}x$ c) \$160
13. a) $P(x) = R(x) - C(x)$

    $= x\left(55 - \frac{1}{800}x\right) - (75000 + x)$

    $= \frac{-1}{800}x^2 + 54x - 75000$

    b) When 21600 tickets are sold

    c) \$508 200