MATHEMATICS 201-103-RE
Differential Calculus
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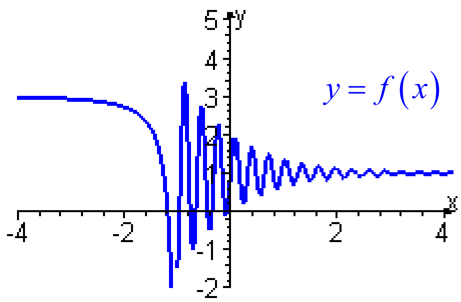
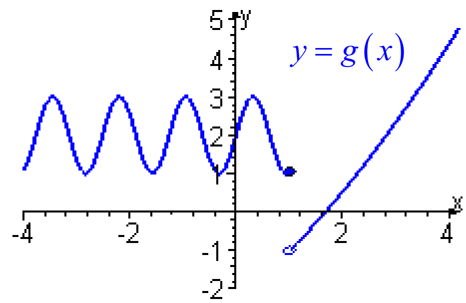
Fall 2011

III – Limits at Infinity

1. For the functions f and g whose graphs are given, state the following.





a) $\lim_{x\to\infty} f(x)$ b) $\lim_{x\to-\infty} f(x)$

c) $\lim_{x\to\infty} g(x)$ d) $\lim_{x\to-\infty} g(x)$

e) The equation of the horizontal asymptotes for f.

f) The equation of the horizontal asymptotes for g.

2. Evaluate the limit

a) $\lim_{x\to\infty} \frac{x+3}{2x-1}$ b) $\lim_{x\to\infty} \frac{x^3-3x+1}{1-4x^3}$ c) $\lim_{x\to-\infty} \frac{x^4-3x^2+1}{2x^4+x}$

d) $\lim_{x\to-\infty} \frac{3-x^3}{x^2+1}$ e) $\lim_{x\to\infty} \frac{x^2-3}{3-4x}$ f) $\lim_{x\to\infty} \frac{x(x+2)}{x^2-4}$

g) $\lim_{x\to\infty} \frac{\sqrt{1+2x^2}}{2x}$ h) $\lim_{x\to-\infty} \frac{\sqrt{1+2x^2}}{2x}$ i) $\lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{x^2}$

j) $\lim_{x\to\infty} \frac{6x}{\sqrt[4]{16x^4+1}}$ k) $\lim_{x\to-\infty} \frac{6x^2}{\sqrt[3]{8x^6+2}}$ l) $\lim_{x\to-\infty} \frac{\sqrt{x^4-x+1}}{x^2-3x}$

m) $\lim_{x\to\infty} \left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)$ n) $\lim_{x\to-\infty} \left(x+\sqrt{x^2+3x}\right)$ o) $\lim_{x\to\infty} \frac{2-x}{\sqrt{9+6x^2}}$

p) $\lim_{x\to-\infty} \frac{2-x}{\sqrt{9+6x^2}}$ q) $\lim_{x\to\infty} \left(x^3-3x+2\right)$ r) $\lim_{x\to-\infty} \left(2-3x-x^4\right)$

s) $\lim_{x\to-\infty} \left(2x^7-3x+1\right)$ t) $\lim_{x\to-\infty} \sqrt{4-3x}$ u) $\lim_{x\to\infty} \left(x^2-\sqrt{x}\right)$

3. Find all horizontal asymptotes (if any) for the following functions.

   a) $f(x)=\dfrac{x^2+4}{x^2-4}$ b) $f(x)=\dfrac{2x^2-3}{x^3-27}$ c) $f(x)=3x^4-x+1$

   d) $f(x)=\dfrac{2\sqrt{x^2+4}}{x-3}$ e) $f(x)=\dfrac{1-3x^2}{\sqrt{x^4-3x}}$ f) $f(x)=\sqrt{x^2+3}-x$

4. Parks Canada introduced 30 elk into a new federal park. The population N of the herd is modeled by

$$N=\frac{10(3+4t)}{1+0.1t}$$

   where t is in years.
   a) Find the size of the herd after 5, 10 and 25 years.
   b) According to this model, what is the limiting size of the herd as time progresses?

5. The cost and revenue functions for a product are $C=34.5x+15000$ and $R=69.9x$.
   a) Find the average profit function $\overline{P}=\dfrac{R-C}{x}$.
   b) What is the limit of the average profit function as x approaches infinity?

6. A company training program has determined that, on the average, a new employee produces $P(s)$ items per day after s days of on-the-job training where

$$P(s)=\frac{75s}{s+8}.$$

   Find and interpret $\lim\limits_{s\to\infty}\dfrac{75s}{s+8}$.

ANSWERS

1.	a) 1	b) 3	c) ∞	d) $\nexists$	f) $y=1,\quad y=3$		g) None
2.	a) $\frac{1}{2}$	b) $\frac{-1}{4}$	c) $\frac{1}{2}$	d) ∞	e) $-\infty$	f) 1	g) $\frac{\sqrt{2}}{2}$
	h) $\frac{-\sqrt{2}}{2}$	i) 0	j) 3	k) 3	l) 1	m) 0	n) $\frac{-3}{2}$
	o) $\frac{-\sqrt{6}}{6}$	p) $\frac{\sqrt{6}}{6}$	q) ∞	r) $-\infty$	s) $-\infty$	t) ∞	u) ∞
3.	a) $y=1$		b) $y=0$		c) None		
	d) $y=2,\quad y=-2$		e) $y=-3$		f) $y=0$		

4. a) 153, 215 and 294 b) 400 elks

5. a) $\overline{P}=35.4-\frac{15000}{x}$ b) 35.4

6. 75 items; the number of items a new employee produces gets closer and closer to 75 as the number of days of training increases.