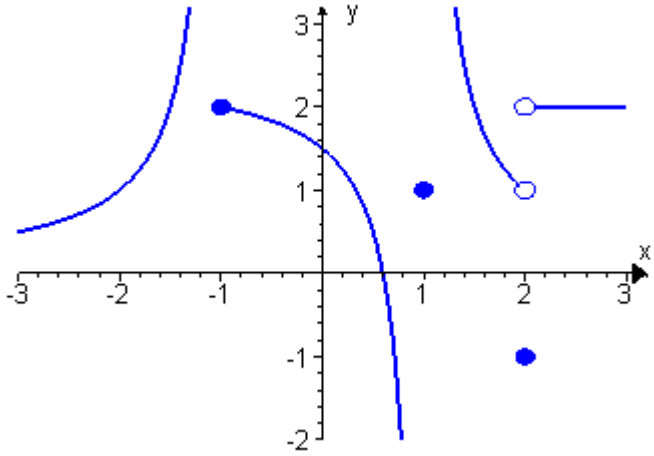
**MATHEMATICS 201-103-RE**
Differential Calculus
Martin Huard
Fall 2011

# II – Infinite Limits

1. For the function $f$ whose graph is given, find the value of the following limits (or function), or explain why it does not exist.




a) $\lim_{x\to -1^+} f(x)$ b) $\lim_{x\to -1^-} f(x)$ c) $\lim_{x\to -1} f(x)$ d) $f(-1)$

e) $\lim_{x\to 1^-} f(x)$ f) $\lim_{x\to 1^+} f(x)$ g) $\lim_{x\to 1} f(x)$ h) $f(1)$

i) $\lim_{x\to 2^-} f(x)$ j) $\lim_{x\to 2^+} f(x)$ k) $\lim_{x\to 2} f(x)$ l) $f(2)$

2. Evaluate the following limits.

a) $\lim_{x\to 2^-} \dfrac{x^2+1}{x-2}$ b) $\lim_{x\to 0^+} \dfrac{\sqrt{4+3x^2}}{5x}$ c) $\lim_{x\to 0^-} \dfrac{\sqrt{4+3x^2}}{5x}$

d) $\lim_{x\to -2^-} \dfrac{x+2}{x^2-4}$ e) $\lim_{x\to 3^+} \dfrac{x^2+5x+1}{x^2-2x-3}$ f) $\lim_{x\to 2^-} \dfrac{x^2+4x+4}{x-2}$

g) $\lim_{x\to 1^-} \dfrac{x^3+1}{x^2-1}$ h) $\lim_{x\to 1^+} \dfrac{x+x^2}{x^2+2x+1}$ i) $\lim_{x\to 3^-} \dfrac{x^2-x-6}{x-3}$

j) $\lim_{x\to 1} \dfrac{x+2}{x^2-2x+1}$ k) $\lim_{x\to 3} \dfrac{x^2-4}{x^2-4x+3}$ l) $\lim_{x\to 0^-} \left(\dfrac{1}{x}-\dfrac{2}{x^3+x}\right)$

3. Find all vertical asymptotes.

a) $f(x)=\dfrac{x^2+1}{x-1}$ b) $f(x)=\dfrac{x^2-1}{x-1}$ c) $f(x)=\dfrac{x^3-8}{x^3+2x^2-2x-4}$

d) $f(x)=x-4-\dfrac{2}{x-4}$ e) $f(x)=\dfrac{3}{\sqrt{x+1}}$ f) $f(x)=\dfrac{x^2+2}{x^3+x}$

4. The cost (in dollars) of removing $p$% of the pollutants from the water in a small lake is given by

$$C = \frac{25000p}{100 - p} \qquad 0 \le p < 100$$

a) Find the cost for removing 50% of the pollutants.
b) What percent of the pollutants can be removed for \$100 000?
c) Evaluate $\lim_{p \to 100^-} C$. Explain your results.

## ANSWERS

1. a) 2 b) $\infty$ c) $\nexists$ d) 2 e) $-\infty$ f) $\infty$ g) $\nexists$
   h) 1 i) 1 j) 2 k) $\nexists$ l) $-1$
2. a) $-\infty$ b) $\infty$ c) $-\infty$ d) $-\frac{1}{4}$ e) $\infty$ f) $-\infty$ g) $-\infty$
   h) $\frac{1}{2}$ i) 5 j) $\infty$ k) $\nexists$ l) $\infty$
3. a) $x = 1$ b) None c) $x = -2$, $x = \sqrt{2}$, $x = -\sqrt{2}$
   d) $x = 4$ e) $x = -1$ f) $x = 0$
4. a) \$25 000 b) 80%
   c) $\infty$ ; The cost function increases without bound as $p$ approaches 100 from the left. Therefore, according to the model, it is not possible to remove 100% of the pollutants.