

I - Limits

1. Evaluate the following limits using a table of values.

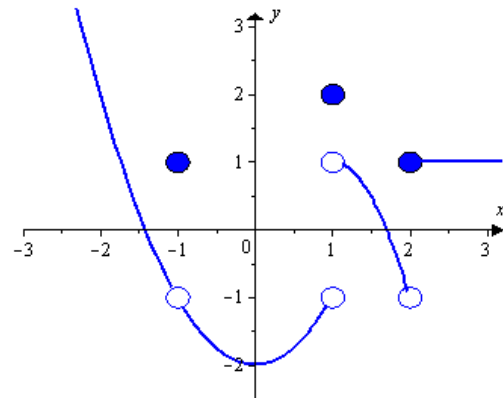
a) $\lim_{x \rightarrow 1} \frac{x-1}{x^3-1}$

b) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$

c) $\lim_{x \rightarrow 2} \frac{x-2}{x^2+2x-8}$

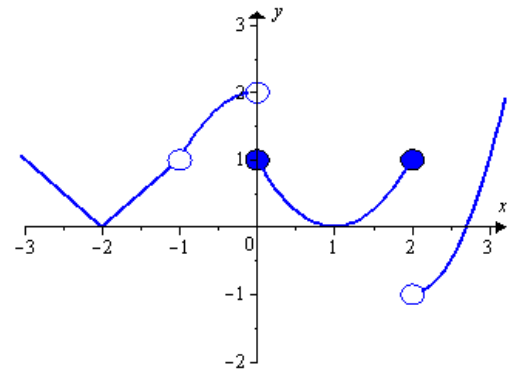
2. For the function f whose graph is given, find the value of the following limits (or function), or explain why it does not exist.

- | | | |
|-------------------------------------|-------------------------------------|------------------------------------|
| a) $\lim_{x \rightarrow -1^+} f(x)$ | b) $\lim_{x \rightarrow -1^-} f(x)$ | c) $\lim_{x \rightarrow -1} f(x)$ |
| d) $f(-1)$ | e) $\lim_{x \rightarrow 1^+} f(x)$ | f) $\lim_{x \rightarrow 1^-} f(x)$ |
| g) $\lim_{x \rightarrow 1} f(x)$ | h) $f(1)$ | i) $\lim_{x \rightarrow -2} f(x)$ |
| j) $\lim_{x \rightarrow 0^-} f(x)$ | k) $\lim_{x \rightarrow 2} f(x)$ | l) $f(2)$ |



3. For the function f whose graph is given, find the value of the following limits (or function), or explain why it does not exist.

- | | | |
|-------------------------------------|-------------------------------------|------------------------------------|
| a) $\lim_{x \rightarrow -1^+} f(x)$ | b) $\lim_{x \rightarrow -1^-} f(x)$ | c) $\lim_{x \rightarrow -1} f(x)$ |
| d) $f(-1)$ | e) $\lim_{x \rightarrow 0^+} f(x)$ | f) $\lim_{x \rightarrow 0^-} f(x)$ |
| g) $\lim_{x \rightarrow 0} f(x)$ | h) $f(0)$ | i) $\lim_{x \rightarrow 2^-} f(x)$ |
| j) $\lim_{x \rightarrow 2^+} f(x)$ | k) $\lim_{x \rightarrow 2} f(x)$ | l) $\lim_{x \rightarrow -2} f(x)$ |



4. Evaluate the following limits, where f is given below.

$$f(x) = \begin{cases} x+2 & x \leq -1 \\ x^3+2 & -1 < x < 2 \\ 3 & x = 2 \\ \sqrt{x-2}+8 & x > 2 \end{cases}$$

- | | | | |
|-------------------------------------|-------------------------------------|------------------------------------|------------|
| a) $\lim_{x \rightarrow -1^+} f(x)$ | b) $\lim_{x \rightarrow -1^-} f(x)$ | c) $\lim_{x \rightarrow -1} f(x)$ | d) $f(-1)$ |
| e) $\lim_{x \rightarrow 2^+} f(x)$ | f) $\lim_{x \rightarrow 2^-} f(x)$ | g) $\lim_{x \rightarrow 2} f(x)$ | h) $f(2)$ |
| i) $\lim_{x \rightarrow -2} f(x)$ | j) $\lim_{x \rightarrow 1^-} f(x)$ | k) $\lim_{x \rightarrow 1^+} f(x)$ | l) $f(1)$ |

5. Given that

$$\lim_{x \rightarrow a} f(x) = 2 \quad \lim_{x \rightarrow a} g(x) = -3 \quad \lim_{x \rightarrow a} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why.

$$\begin{array}{lll} \text{a) } \lim_{x \rightarrow a} [f(x) + g(x)] & \text{b) } \lim_{x \rightarrow a} [f(x)]^5 & \text{c) } \lim_{x \rightarrow a} \frac{1}{f(x)} \\ \text{d) } \lim_{x \rightarrow a} f(x)g(x) & \text{e) } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} & \text{f) } \lim_{x \rightarrow a} \frac{f(x)}{h(x)} \\ \text{g) } \lim_{x \rightarrow a} \frac{h(x)}{f(x)} & \text{h) } \lim_{x \rightarrow a} \frac{4f(x)}{g(x) - h(x)} & \text{i) } \lim_{x \rightarrow a} \frac{f(x) - 1}{1 - f(x)} \end{array}$$

6. Evaluate the following limits.

$$\begin{array}{lll} \text{a) } \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3} & \text{b) } \lim_{x \rightarrow 2} \frac{x^3 + 3x^2 - 10x}{2x^2 - 5x + 2} & \text{c) } \lim_{x \rightarrow -3} \frac{x^2 - 9}{x^2 + 9} \\ \text{d) } \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 + 3x - 4} & \text{e) } \lim_{x \rightarrow 25} \frac{x - 25}{\sqrt{x} - 5} & \text{f) } \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{x - 2} \\ \text{g) } \lim_{x \rightarrow 2} \frac{\frac{2}{x} - 1}{x - 2} & \text{h) } \lim_{x \rightarrow 3} \frac{\frac{1}{3} - \frac{1}{x}}{x^2 + 2x - 15} & \text{i) } \lim_{x \rightarrow \frac{5}{2}} \frac{4x^2 - 25}{5 - 2x} \\ \text{j) } \lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h} & \text{k) } \lim_{y \rightarrow 0} \frac{2 - \sqrt{4-y}}{y} & \text{l) } \lim_{x \rightarrow 1} \frac{\frac{1}{\sqrt{x}} - 1}{1 - x} \\ \text{m) } \lim_{x \rightarrow 1} \frac{1 - x}{2 - \sqrt{x^2 + 3}} & \text{n) } \lim_{x \rightarrow -2^+} \frac{2 + t}{\sqrt{4 - t^2}} & \text{o) } \lim_{x \rightarrow 2} \frac{x^4 - 16}{x^2 - 2x} \\ \text{p) } \lim_{x \rightarrow 2} \frac{(x-2)^3}{x^2 - 4} & \text{q) } \lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{5}{x^2 - x - 6} \right) & \text{r) } \lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) \\ \text{s) } \lim_{x \rightarrow 4} \frac{t^2 - 4t}{t^4 - 16} & \text{t) } \lim_{x \rightarrow 7} \frac{\sqrt{43-x} - 2\sqrt{x+2}}{x^2 - 8x + 7} & \text{u) } \lim_{x \rightarrow -3} \frac{x^3 + 3x^2 - 4x - 12}{x^3 + 27} \\ \text{v) } \lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{x}{x+2}}{3 - \frac{x+4}{x}} & \text{w) } \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{\sqrt{x-3} - 1} & \text{x) } \lim_{x \rightarrow -1} \frac{\frac{1}{\sqrt{x+5}} - \frac{1}{\sqrt{3-x}}}{x+1} \\ \text{y) } \lim_{x \rightarrow 3} \frac{\frac{4}{x+5} - \frac{x}{3x-3}}{\frac{3}{x^2} - \frac{1}{x}} & \text{z) } \lim_{x \rightarrow 4} \frac{x^3 - 5x^2 - 2x + 24}{x^3 - 3x^2 - 3x - 4} & \text{aa) } \lim_{x \rightarrow -2} \frac{\frac{3}{x^2+4} - \frac{x-1}{x^3}}{\frac{x^3}{x^2-2} + 4} \end{array}$$

7. Evaluate the following limits.

$$\begin{array}{lll} \text{a) } \lim_{x \rightarrow a} \frac{x^2 - a^2}{x - a} & \text{b) } \lim_{t \rightarrow a} \frac{\frac{1}{t+2} - \frac{1}{a+2}}{a^2 - t^2} & \text{c) } \lim_{t \rightarrow a} \frac{\sqrt{t+3} - \sqrt{a+3}}{t - a} \end{array}$$

8. Evaluate the limit (if it exists)

$$\text{a) } \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

$$\text{b) } \lim_{x \rightarrow 3} \frac{x^2 - 9}{|x - 3|}$$

$$\text{c) } \lim_{x \rightarrow 3} \left| \frac{x^2 - 9}{x - 3} \right|$$

$$\text{d) } \lim_{x \rightarrow -2} \frac{|x + 2|}{x + 2}$$

$$\text{e) } \lim_{x \rightarrow 2} \frac{|x| - 2}{x - 2}$$

$$\text{f) } \lim_{x \rightarrow 4^+} \frac{x - 4}{\sqrt{x - 4}}$$

$$\text{g) } \lim_{x \rightarrow 3} f(x) \quad f(x) = \begin{cases} x + 2 & x < 3 \\ x^2 - 5 & x \geq 3 \end{cases}$$

$$\text{h) } \lim_{x \rightarrow -1} f(x) \quad f(x) = \begin{cases} \frac{1}{x} & x < -1 \\ 5 & x = -1 \\ x^2 - 2 & x > -1 \end{cases}$$

$$\text{i) } \lim_{x \rightarrow 5} f(x) \quad f(x) = \begin{cases} \frac{\frac{7}{x+2} - \frac{2}{x-3}}{x^3 - 125} & x < 5 \\ \sqrt{x+4} - 3 & \\ \frac{7x^2 - 35x}{7x^2 - 35x} & x \geq 5 \end{cases}$$

9. The Ace Parking Garages charges \$200 for parking for 2 hours or less, and 50 cents for each extra hour or part of an hour after the 2-hour minimum. The parking charges for the first 5 hours could be written as a function of time as follows:

$$f(t) = \begin{cases} \$2.00 & \text{if } 0 < t \leq 2 \\ \$2.50 & \text{if } 2 < t \leq 3 \\ \$3.00 & \text{if } 3 < t \leq 4 \\ \$3.50 & \text{if } 4 < t \leq 5 \end{cases}$$

Find

$$\text{a) } \lim_{t \rightarrow 1} f(t)$$

$$\text{b) } \lim_{t \rightarrow 2} f(t)$$

ANSWERS

1. a) $\frac{1}{3}$ b) 0 c) $\frac{1}{6}$
2. a) -1 b) -1 c) -1 d) 1 e) 1 f) -1 g) $\cancel{\neq}$
 h) 2 i) 2 j) -2 k) $\cancel{\neq}$ l) 1
3. a) 1 b) 1 c) 1 d) $\cancel{\neq}$ e) 1 f) 2 g) $\cancel{\neq}$
 h) 1 i) 1 j) -1 k) $\cancel{\neq}$ l) 0
4. a) 1 b) 1 c) 1 d) 1 e) 8 f) 10 g) $\cancel{\neq}$
 h) 3 i) 0 j) 3 k) 3 l) 3
5. a) -1 b) 32 c) $\frac{1}{2}$ d) -6 e) $\frac{-2}{3}$ f) $\cancel{\neq}$ g) 0
 h) $\frac{-8}{3}$ i) -1
6. a) 5 b) $\frac{14}{3}$ c) 0 d) $\frac{3}{5}$ e) 10 f) $\frac{1}{4}$ g) $\frac{-1}{2}$
 h) $\frac{1}{72}$ i) -10 j) 6 k) $\frac{1}{4}$ l) $\frac{1}{2}$ m) 2 n) 0
 o) 16 p) 0 q) $\frac{1}{5}$ r) $\frac{-1}{2}$ s) 0 t) $\frac{5}{72}$ u) $\frac{5}{27}$
 v) $\frac{-3}{8}$ w) $\frac{1}{2}$ x) $\frac{-1}{8}$ y) $\frac{-3}{16}$ z) $\frac{2}{7}$ aa) $\frac{1}{8}$
7. a) $2a$ b) $\frac{1}{2a(a+2)^2}$ c) $\frac{1}{2\sqrt{a+3}}$
8. a) 6 b) $\cancel{\neq}$ c) 6 d) 0 e) $\cancel{\neq}$ f) 1 g) $\cancel{\neq}$
 h) -1 i) $\frac{1}{210}$
9. a) \$2.00 b) $\lim_{x \rightarrow 2} f(t) \cancel{\neq}$