MATHEMATICS 201-009-50
Precalculus
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XXII –Trigonometric Equations

1. Find all solutions to the following equations.

a) $\sin\theta = -\frac{\sqrt{2}}{2}$
b) $\cos\theta = \frac{1}{2}$
c) $\tan\theta = -1$
d) $\sec\theta = \sqrt{2}$
e) $\csc\theta = \frac{2\sqrt{3}}{3}$
f) $\cot\theta = 0$
g) $\sin 3\theta = \frac{\sqrt{3}}{2}$
h) $\cos\frac{3\theta}{2} = 1$
i) $\sec\frac{\theta}{4} = -2$
j) $\sin\left(\frac{\theta}{2} + \frac{\pi}{4}\right) = -1$
k) $\cos\left(3\theta - \frac{\pi}{2}\right) = \frac{-\sqrt{3}}{2}$
l) $\tan\left(2\theta - \frac{\pi}{6}\right) = \sqrt{3}$

2. Find all solutions to the given equation on the interval $[0, 2\pi)$.

a) $\cos\left(2\theta + \frac{5\pi}{4}\right) = \frac{\sqrt{2}}{2}$
b) $\csc(5\theta - \pi) = 2$
c) $\cos^2\theta - 1 = 0$
d) $2\sin^2\theta - \sin\theta - 1 = 0$
e) $2\cos^2\theta + \cos\theta - 1 = 0$
f) $\cos\theta = \sec\theta$
g) $\tan\theta = \cot\theta$
h) $2\sin\theta\cos\theta + \cos\theta = 0$
i) $\left(2\cos x + \sqrt{3}\right)(2\sin x - 1) = 0$
j) $\tan\theta\sin\theta + \sin\theta = 0$
k) $\sin^2\theta = 2\sin\theta + 3$
l) $\sin 2\theta = \tan 2\theta$
m) $\cos 2\theta + 5\cos\theta + 3 = 0$
n) $\sec^2\theta + \tan\theta = 1$
o) $\cos 4\theta + \cos 6\theta = 0$
p) $2\sin^2\theta = 2 + \cos\theta$
q) $\cos\theta + \sin\theta\tan\theta = 2$
r) $\sec\theta\csc\theta = 2\csc\theta$
s) $\sin 2\theta + \sin 4\theta = 0$
t) $\csc^2\theta = \cot\theta + 1$
u) $\tan 3x\tan x - \tan 3x = 0$
v) $3\tan^3 x - 3\tan^2 x - \tan x + 1 = 0$
w) $\cos^2 2\theta - \sin^2 2\theta = 1$
x) $4\sin x\cos x + 2\sin x - 2\cos x - 1 = 0$
y) $3\sec^2\theta + 4\cos^2\theta = 7$
z) $\sin 2x - \cos x = 0$
aa) $\tan x + \cot x = 4\sin 2x$
bb) $\cos x\cos 2x + \sin x\sin 2x = \frac{1}{2}$
cc) $\sin 7x - \sin 3x = \cos 5x$
dd) $\sin 3x\cos x - \cos 3x\sin x = \frac{\sqrt{3}}{2}$

ANSWERS

1. a) $\frac{-\pi}{4}+2\pi n,\ n\in\mathbb{Z}$ and $\frac{5\pi}{4}+2\pi n,\ n\in\mathbb{Z}$
   b) $\frac{\pi}{3}+2\pi n,\ n\in\mathbb{Z}$ and $\frac{5\pi}{3}+2\pi n,\ n\in\mathbb{Z}$
   c) $\frac{3\pi}{4}+\pi n,\ n\in\mathbb{Z}$
   d) $\frac{\pi}{4}+2\pi n,\ n\in\mathbb{Z}$ and $\frac{5\pi}{4}+2\pi n,\ n\in\mathbb{Z}$
   e) $\frac{\pi}{3}+2\pi n,\ n\in\mathbb{Z}$ and $\frac{2\pi}{3}+2\pi n,\ n\in\mathbb{Z}$
   f) $\frac{\pi}{2}+\pi n,\ n\in\mathbb{Z}$
   g) $\frac{\pi}{9}+\frac{2\pi}{3}n,\ n\in\mathbb{Z}$ and $\frac{2\pi}{9}+\frac{2\pi}{3}n,\ n\in\mathbb{Z}$
   h) $\frac{4\pi}{3}n,\ n\in\mathbb{Z}$
   i) $\frac{8\pi}{3}+8\pi n,\ n\in\mathbb{Z}$ and $\frac{16\pi}{3}+8\pi n,\ n\in\mathbb{Z}$
   j) $\frac{5\pi}{2}+4\pi n,\ n\in\mathbb{Z}$
   k) $\frac{4\pi}{9}+\frac{2\pi}{3}n,\ n\in\mathbb{Z}$ and $\frac{5\pi}{9}+\frac{2\pi}{3}n,\ n\in\mathbb{Z}$
   l) $\frac{\pi}{4}+\frac{\pi}{2}n,\ n\in\mathbb{Z}$
2. a) $\frac{\pi}{4},\frac{\pi}{2},\frac{5\pi}{4},\frac{3\pi}{2}$
   b) $\frac{7\pi}{30},\frac{11\pi}{30},\frac{19\pi}{30},\frac{23\pi}{30},\frac{31\pi}{30},\frac{7\pi}{6},\frac{43\pi}{30},\frac{47\pi}{30},\frac{11\pi}{6},\frac{59\pi}{30}$
   c) $0,\pi$
   d) $\frac{\pi}{2},\frac{7\pi}{6},\frac{11\pi}{6}$
   e) $\frac{\pi}{3},\pi,\frac{5\pi}{3}$
   f) $0,\pi$
   g) $\frac{\pi}{4},\frac{5\pi}{4}$
   h) $\frac{\pi}{2},\frac{7\pi}{6},\frac{3\pi}{2},\frac{11\pi}{6}$
   i) $\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6}$
   j) $0,\frac{3\pi}{4},\pi,\frac{7\pi}{4}$
   k) $\frac{3\pi}{2}$
   l) $0,\frac{\pi}{2},\pi,\frac{3\pi}{2}$
   m) $\frac{2\pi}{3},\frac{4\pi}{3}$
   n) $0,\frac{3\pi}{4},\pi,\frac{7\pi}{4}$
   o) $\frac{\pi}{10},\frac{\pi}{2},\frac{9\pi}{10},\frac{11\pi}{10},\frac{3\pi}{2},\frac{19\pi}{10}$
   p) $\frac{\pi}{2},\frac{2\pi}{3},\frac{4\pi}{3},\frac{3\pi}{2}$
   q) $\frac{\pi}{3},\frac{5\pi}{3}$
   r) $\frac{\pi}{3},\frac{5\pi}{3}$
   s) $0,\frac{\pi}{3},\frac{\pi}{2},\frac{2\pi}{3},\pi,\frac{4\pi}{3},\frac{3\pi}{2},\frac{5\pi}{3}$
   t) $\frac{\pi}{4},\frac{\pi}{2},\frac{5\pi}{4},\frac{3\pi}{2}$
   u) $0,\frac{\pi}{4},\frac{\pi}{3},\frac{2\pi}{3},\pi,\frac{5\pi}{4},\frac{4\pi}{3},\frac{5\pi}{3}$
   v) $\frac{\pi}{6},\frac{\pi}{4},\frac{5\pi}{6},\frac{7\pi}{6},\frac{5\pi}{4},\frac{11\pi}{6}$
   w) $0,\frac{\pi}{2},\pi,\frac{3\pi}{2}$
   x) $\frac{\pi}{6},\frac{2\pi}{3},\frac{5\pi}{6},\frac{4\pi}{3}$
   y) $0,\frac{\pi}{6},\frac{5\pi}{6},\pi,\frac{7\pi}{6},\frac{11\pi}{6}$
   z) $\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6},\frac{3\pi}{2}$
   aa) $\frac{\pi}{8},\frac{3\pi}{8},\frac{5\pi}{8},\frac{7\pi}{8},\frac{9\pi}{8},\frac{11\pi}{8},\frac{13\pi}{8},\frac{15\pi}{8}$
   bb) $\frac{\pi}{3},\frac{5\pi}{3}$
   cc) $\frac{\pi}{12},\frac{\pi}{10},\frac{3\pi}{10},\frac{5\pi}{12},\frac{5\pi}{10},\frac{7\pi}{10},\frac{9\pi}{10},\frac{11\pi}{10},\frac{13\pi}{12},\frac{13\pi}{10},\frac{15\pi}{10},\frac{17\pi}{12},\frac{17\pi}{10},\frac{19\pi}{10}$
   dd) $\frac{\pi}{6},\frac{\pi}{3},\frac{7\pi}{6},\frac{4\pi}{3}$