

MATHEMATICS 201-009-50

Precalculus
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Fall 2007

XVIII – Trigonometric Graphs

1. Find the amplitude, period, and phase shift of the function, and use that information to sketch its graph.

a) $f(x) = 2 + \cos x$

b) $f(x) = 3 \sin\left(x + \frac{\pi}{4}\right)$

c) $f(x) = 4 - 2 \sin x$

d) $f(x) = 2 \sin\left(x - \frac{\pi}{6}\right)$

e) $f(x) = 3 \cos\left(x + \frac{\pi}{3}\right)$

f) $f(x) = \sin 2\left(x - \frac{\pi}{4}\right)$

g) $f(x) = -2 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right)$

h) $f(x) = 3 \cos(2x + \pi)$

i) $f(x) = \cos(3x - \pi)$

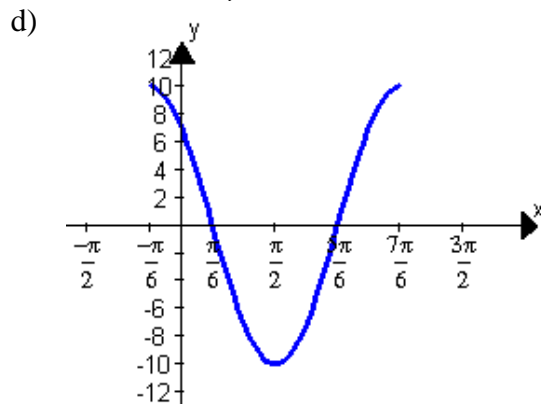
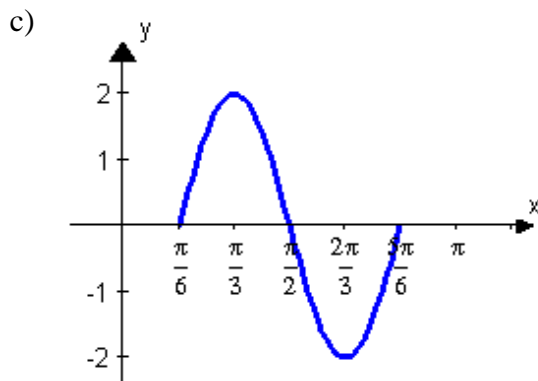
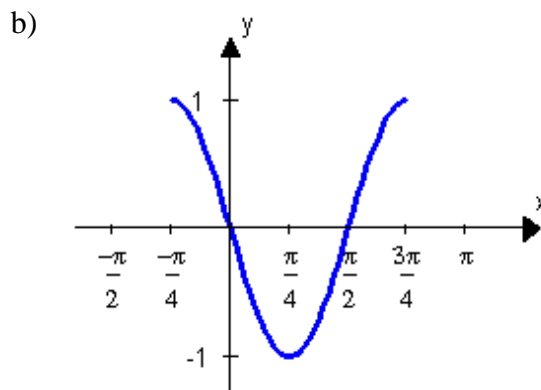
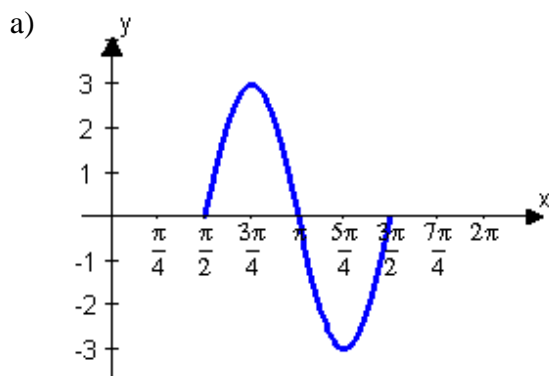
j) $f(x) = -2 \sin\left(\frac{x}{3} - \frac{\pi}{6}\right)$

k) $f(x) = \cos \pi(x - 1)$

l) $f(x) = 2 \sin \frac{\pi}{2}(x + 2)$

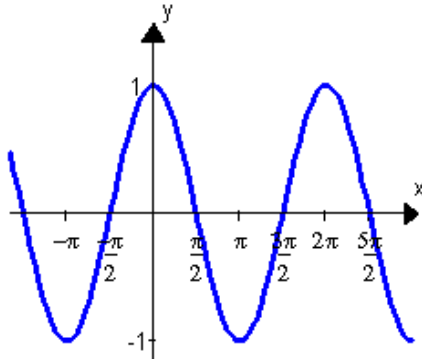
2. A complete period for a sine or cosine curve is given. Find the amplitude, period, and phase shift, and write the equation for the function in the form $f(x) = a \sin k(x - b)$ or

$f(x) = a \cos k(x - b)$.

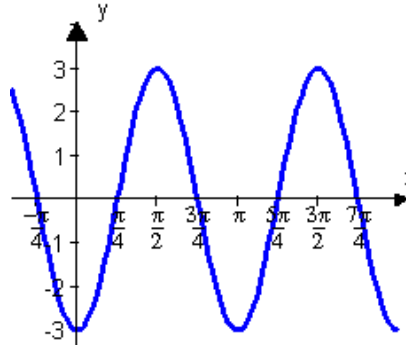


3. Find the equation of the function whose graph is given below. Give your answer in the form $f(x) = a \sin k(x - b)$. Note: More than one answer is possible.

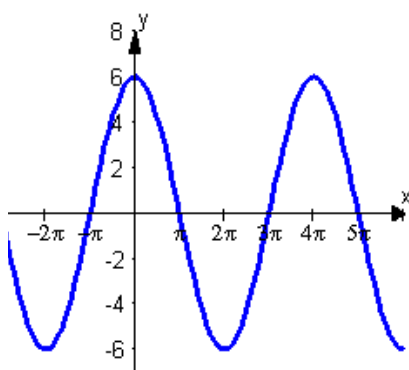
a)



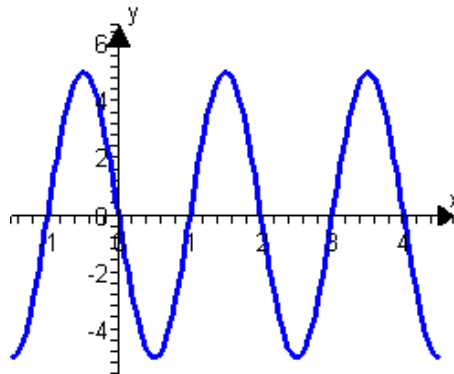
b)



c)

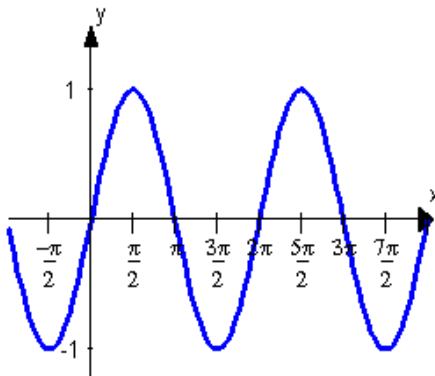


d)

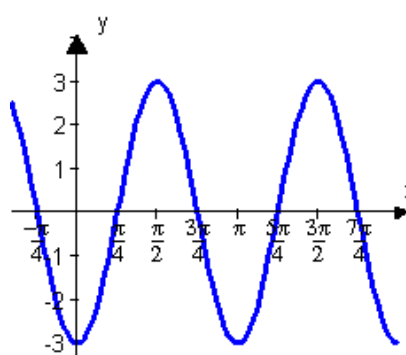


4. Find the equation of the function whose graph is given below. Give your answer in the form $f(x) = a \cos k(x - b)$. Note: More than one answer is possible.

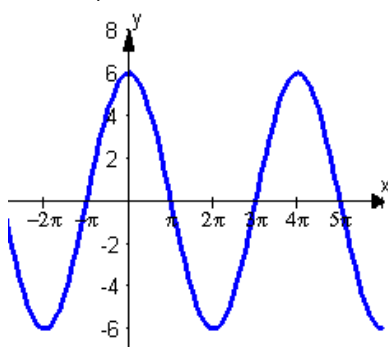
a)



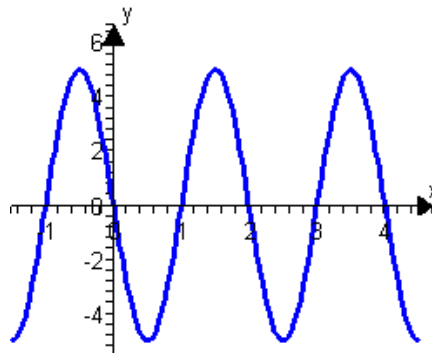
b)



c)



d)



5. Sketch a graph of the function given below. Find the period and phase shift.

a) $f(x) = -\tan x$

b) $f(x) = \tan\left(x + \frac{\pi}{4}\right)$

c) $f(x) = 3 \tan x$

d) $f(x) = -2 \cot x$

e) $f(x) = \cot\left(x + \frac{2\pi}{3}\right)$

f) $f(x) = \tan 2x$

g) $f(x) = -\tan \frac{x}{2}$

h) $f(x) = \cot\left(\frac{\pi}{2}x\right)$

i) $f(x) = \cot 2\left(x - \frac{\pi}{4}\right)$

j) $f(x) = \tan 3\left(x - \frac{\pi}{6}\right)$

k) $f(x) = -\tan(2x + \pi)$

l) $f(x) = \cot\left(2x - \frac{\pi}{2}\right)$

m) $f(x) = 2 \csc x - 3$

n) $f(x) = -\sec x + 1$

o) $f(x) = 2 \csc 3x$

p) $f(x) = 2 \sec\left(x - \frac{\pi}{3}\right)$

q) $f(x) = \csc\left(x + \frac{\pi}{3}\right) + 1$

r) $f(x) = 3 \csc 2\left(x + \frac{\pi}{4}\right)$

s) $f(x) = -2 \sec(2x + \pi)$

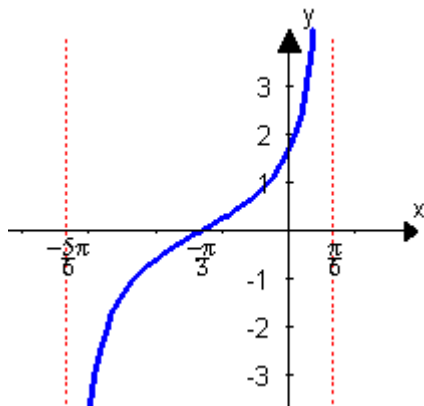
t) $f(x) = 4 \sec 3\left(x - \frac{\pi}{2}\right)$

u) $f(x) = -5 \csc\left(3x + \frac{\pi}{2}\right)$

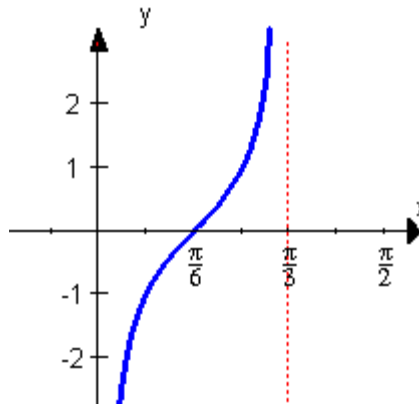
v) $f(x) = 3 \sec(2x + \pi) - 3$

6. A complete period for a trigonometric function is given. Find an equation for the curve.

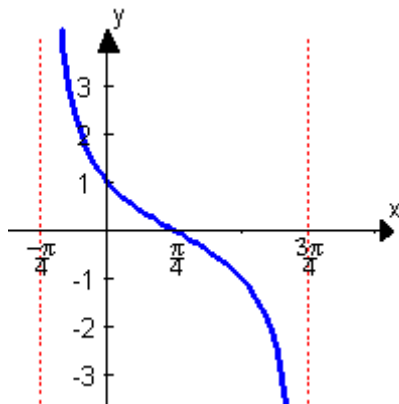
a)



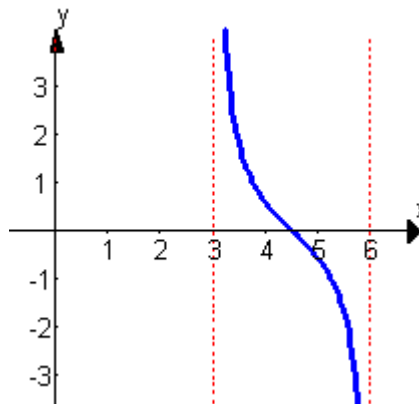
b)

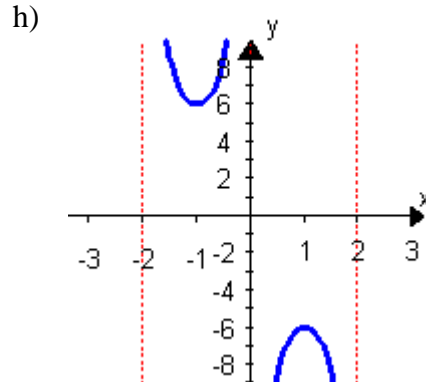
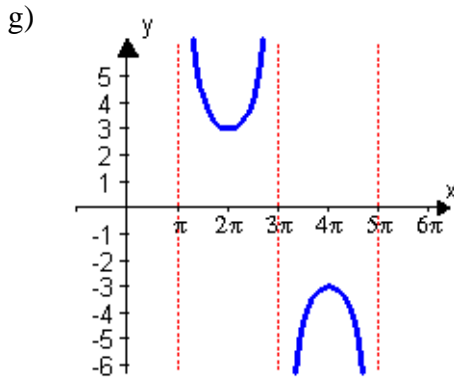
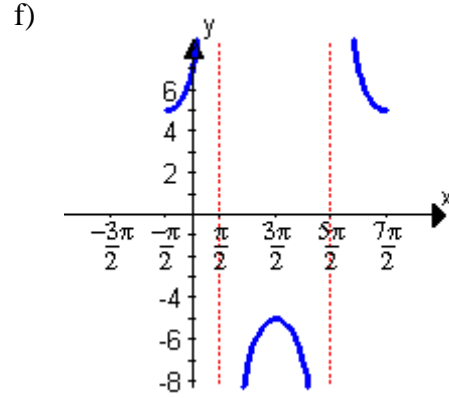
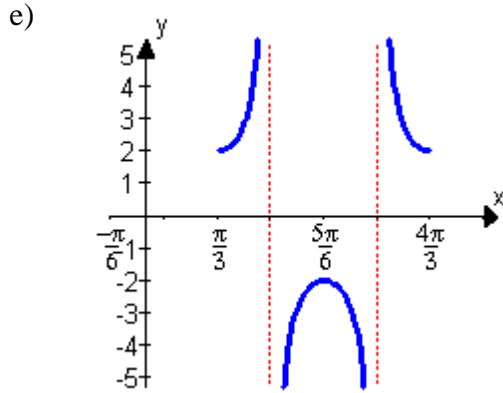


c)

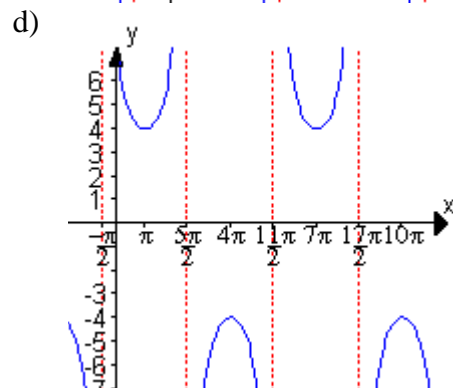
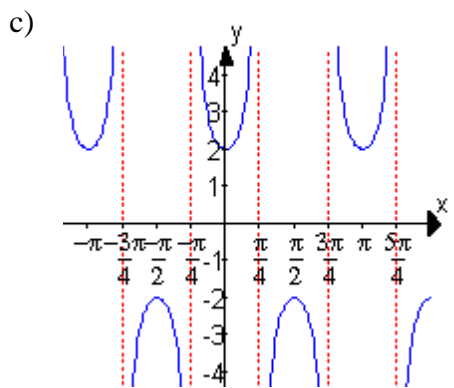
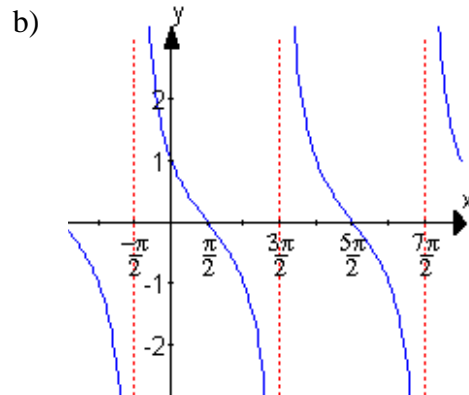
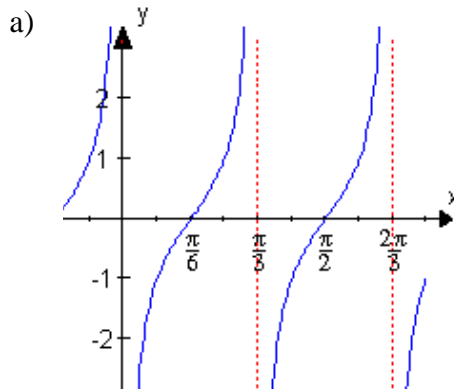


d)



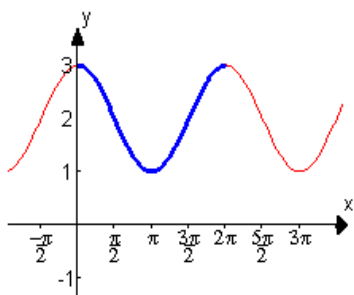


7. Find an equation for the function whose graph is given. Note: More than one answer is possible.

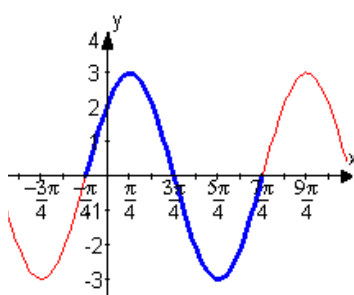


ANSWERS

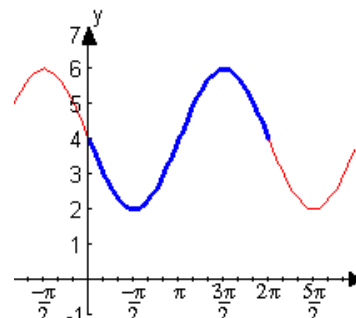
1. a) Amplitude: 1
 Period: 2π
 Phase Shift: 0



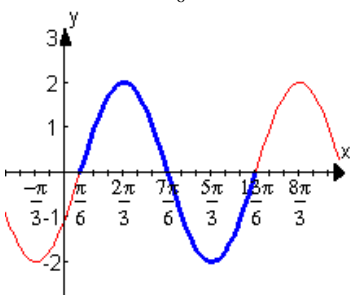
- b) Amplitude: 3
 Period: 2π
 Phase Shift: $-\frac{\pi}{4}$



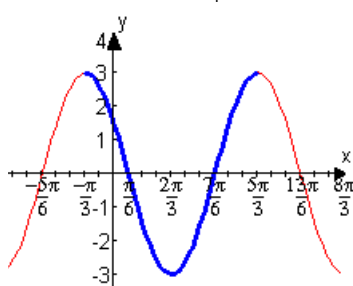
- c) Amplitude: 2
 Period: 2π
 Phase Shift: 0



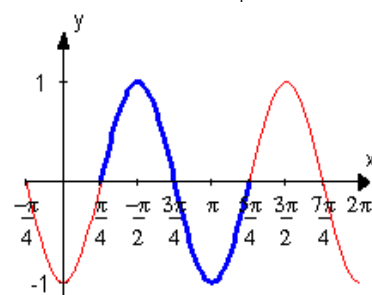
- d) Amplitude: 2
 Period: 2π
 Phase Shift: $\frac{\pi}{6}$



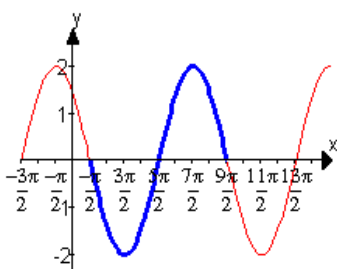
- e) Amplitude: 3
 Period: 2π
 Phase Shift: $-\frac{\pi}{4}$



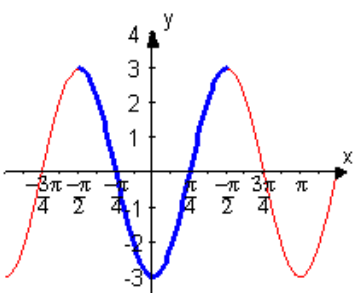
- f) Amplitude: 1
 Period: π
 Phase Shift: $\frac{\pi}{4}$



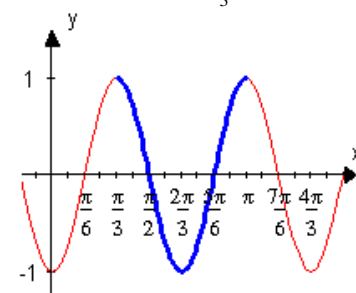
- g) Amplitude: 1
 Period: 2π
 Phase Shift: 0



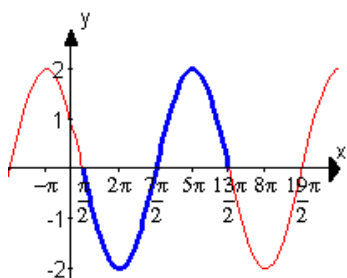
- h) Amplitude: 3
 Period: π
 Phase Shift: $-\frac{\pi}{2}$



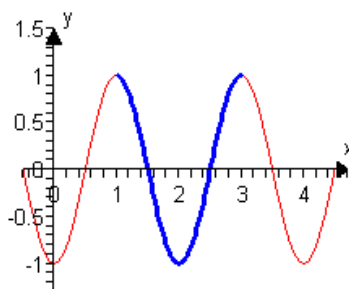
- i) Amplitude: 1
 Period: $\frac{2\pi}{3}$
 Phase Shift: $\frac{\pi}{3}$



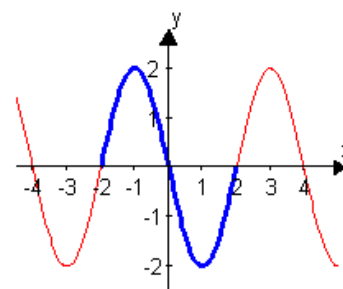
- j) Amplitude: 2
 Period: 6π
 Phase Shift: $\frac{\pi}{2}$



- k) Amplitude: 3
 Period: 2
 Phase Shift: 1



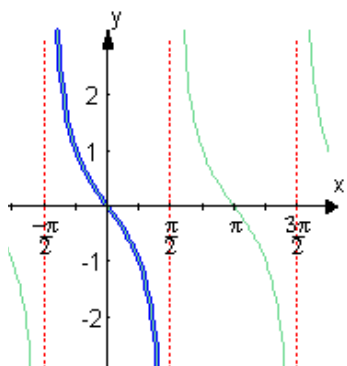
- l) Amplitude: 2
 Period: 4
 Phase Shift: -2



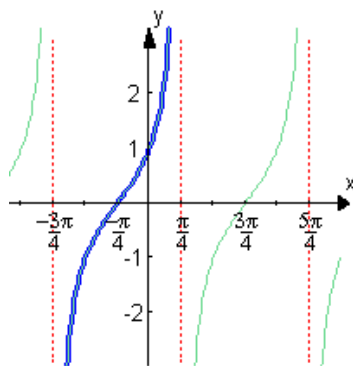
2. a) $f(x) = 3\sin 2(x - \frac{\pi}{2})$
 c) $f(x) = 2\sin 3(x - \frac{\pi}{6})$
 3. a) $f(x) = \sin(x + \frac{\pi}{2})$
 c) $f(x) = 6\sin \frac{1}{2}(x + \pi)$
 4. a) $f(x) = \cos(x - \frac{\pi}{2})$
 c) $f(x) = 6\sin \frac{1}{2}(x + \pi)$

- b) $f(x) = \cos 2(x + \frac{\pi}{4})$
 d) $f(x) = 10\cos \frac{3}{2}(x + \frac{\pi}{6})$
 b) $f(x) = 3\sin 2(x - \frac{\pi}{4})$
 d) $f(x) = 5\sin \pi(x - 1)$
 b) $f(x) = 3\sin 2(x - \frac{\pi}{4})$
 d) $f(x) = 5\sin \pi(x - 1)$

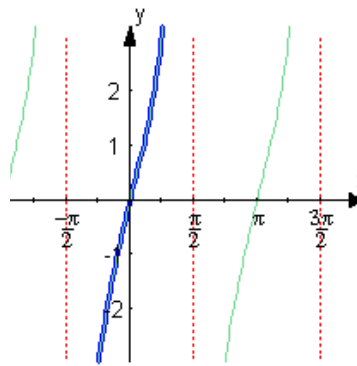
5. a) Period: π
 Phase Shift: 0



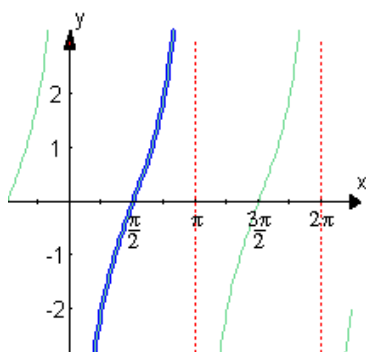
- b) Period: π
 Phase Shift: $-\frac{\pi}{4}$



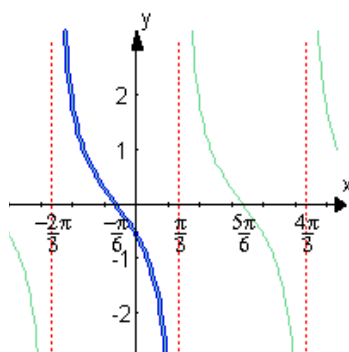
- c) Period: π
 Phase Shift: 0



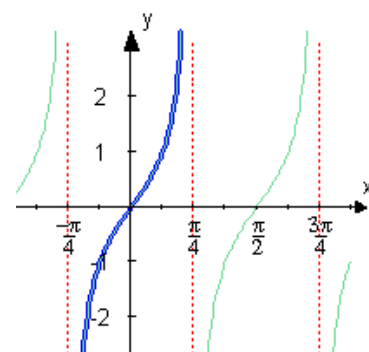
- d) Period: π
 Phase Shift: 0



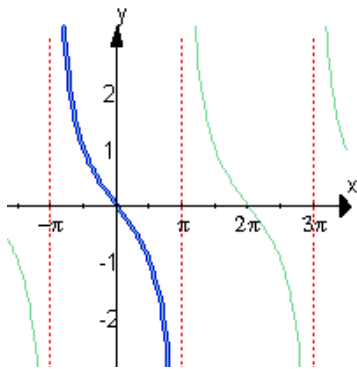
- e) Period: π
 Phase Shift: $-\frac{\pi}{4}$



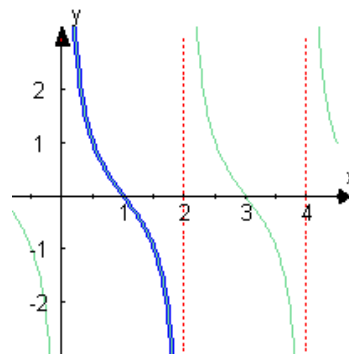
- f) Period: $\frac{\pi}{2}$
 Phase Shift: 0



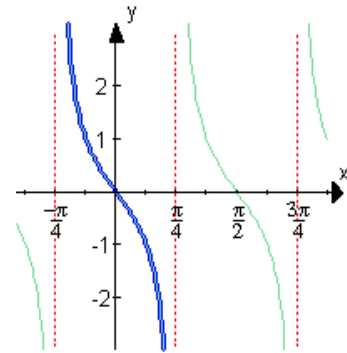
g) Period: 2π
Phase Shift: 0



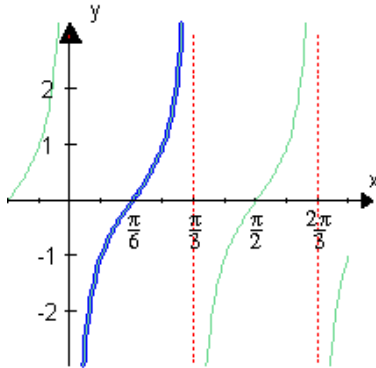
h) Period: 2
Phase Shift: 0



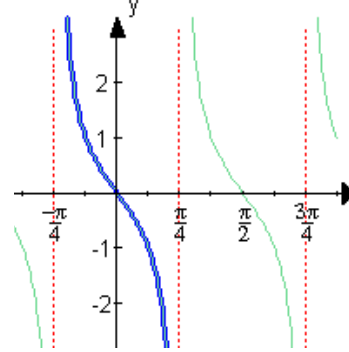
i) Period: $\frac{\pi}{2}$
Phase Shift: $-\frac{\pi}{4}$



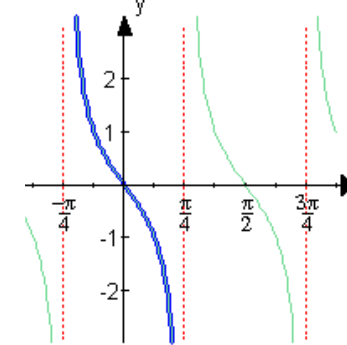
j) Period: $\frac{\pi}{3}$
Phase Shift: $\frac{\pi}{6}$



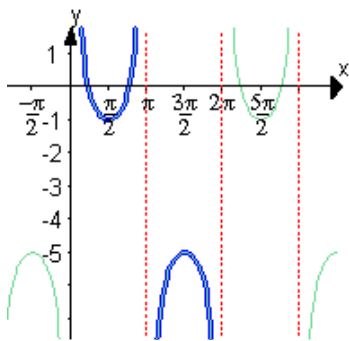
k) Period: $\frac{\pi}{2}$
Phase Shift: 0 or $-\frac{\pi}{2}$



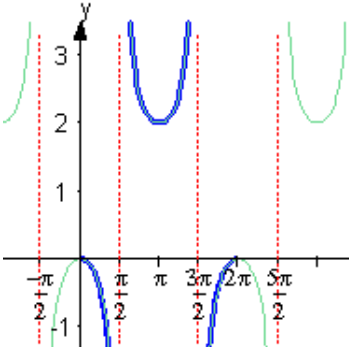
l) Period: $\frac{\pi}{2}$
Phase Shift: $-\frac{\pi}{4}$



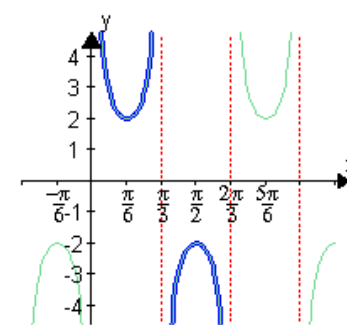
m) Period: 2π
Phase Shift: 0



n) Period: 2π
Phase Shift: 0

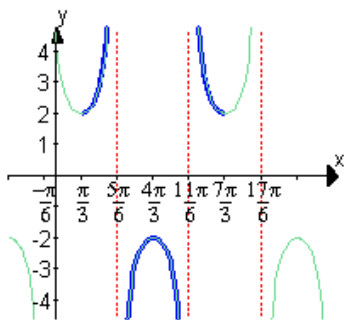


o) Period: $\frac{2\pi}{3}$
Phase Shift: 0



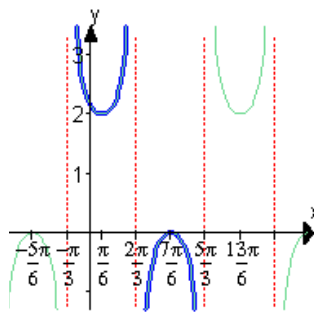
p) Period: 2π

Phase Shift: $\frac{\pi}{3}$



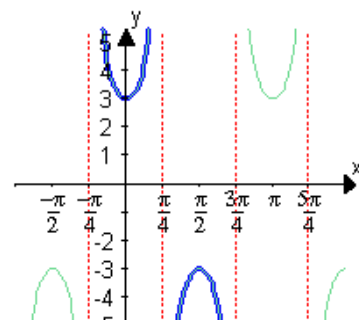
q) Period: 2π

Phase Shift: $-\frac{\pi}{3}$



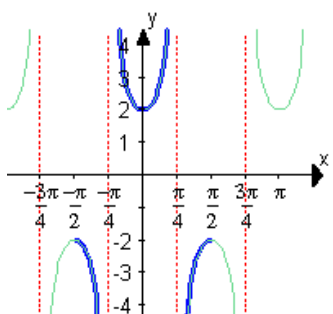
r) Period: π

Phase Shift: $-\frac{\pi}{4}$



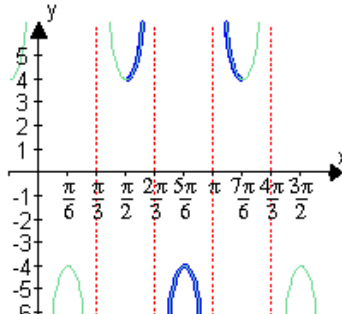
s) Period: π

Phase Shift: $-\frac{\pi}{2}$



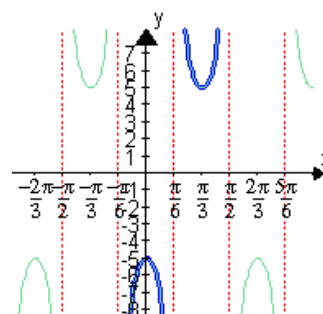
t) Period: $\frac{2\pi}{3}$

Phase Shift: $\frac{\pi}{2}$



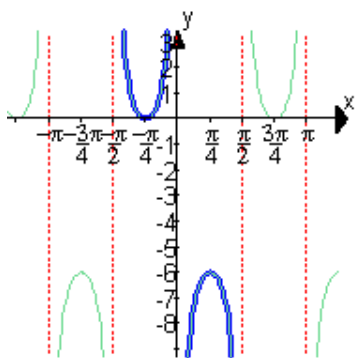
u) Period: $\frac{2\pi}{3}$

Phase Shift: $-\frac{\pi}{6}$



v) Period: π

Phase Shift: $-\frac{\pi}{2}$



6. a) $f(x) = \tan\left(x + \frac{\pi}{3}\right)$

b) $f(x) = \tan 3\left(x - \frac{\pi}{6}\right)$

c) $\cot\left(x + \frac{\pi}{4}\right)$

d) $f(x) = \cot \frac{\pi}{3}\left(x - 3\right)$

e) $f(x) = 2 \sec 2\left(x - \frac{\pi}{3}\right)$

f) $f(x) = 5 \sec \frac{1}{2}\left(x + \frac{\pi}{2}\right)$

g) $f(x) = 3 \csc \frac{1}{2}\left(x - \pi\right)$

h) $f(x) = 5 \csc \frac{\pi}{2}\left(x + 2\right)$

7. a) $f(x) = \tan 3\left(x - \frac{\pi}{6}\right) = -\cot(3x)$

b) $f(x) = \cot \frac{1}{2}\left(x + \frac{\pi}{2}\right) = -\tan \frac{1}{2}\left(x - \frac{\pi}{2}\right)$

c) $f(x) = 2 \sec(2x) = 2 \csc 2\left(x + \frac{\pi}{4}\right)$

d) $f(x) = 4 \csc \frac{1}{3}\left(x + \frac{\pi}{2}\right) = 4 \sec \frac{1}{3}\left(x - \pi\right)$