

MATHEMATICS 201-009-50

Precalculus

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XVII – Trigonometric Functions of Real Numbers

1. Find the exact value for the trigonometric function at the given number (Do not use a calculator!)

a) $\sin\left(\frac{2\pi}{3}\right)$

b) $\cos\frac{3\pi}{4}$

c) $\cos\left(\frac{5\pi}{6}\right)$

d) $\sin\left(\frac{5\pi}{2}\right)$

e) $\sin\frac{11\pi}{6}$

f) $\sin\left(\frac{-4\pi}{3}\right)$

g) $\cos\pi$

h) $\sin 3\pi$

i) $\cos\frac{7\pi}{4}$

j) $\cos\frac{3\pi}{2}$

k) $\tan\frac{\pi}{3}$

l) $\csc\frac{\pi}{6}$

m) $\sec\frac{5\pi}{6}$

n) $\cot\frac{3\pi}{4}$

o) $\tan\left(\frac{-\pi}{4}\right)$

p) $\sec\frac{-3\pi}{4}$

q) $\csc\frac{3\pi}{2}$

r) $\sec\left(\frac{7\pi}{4}\right)$

s) $\csc\left(\frac{7\pi}{6}\right)$

t) $\tan\pi$

u) $\csc\frac{-13\pi}{6}$

v) $\cot\pi$

w) $\sec\frac{5\pi}{3}$

x) $\cot\frac{5\pi}{4}$

2. Find the value of the six trigonometric functions of θ .

a) $\sin\theta = \frac{5}{13}$ and θ is in quadrant II.

b) $\cos\theta = \frac{-3}{5}$ and θ is in quadrant III.

c) $\tan\theta = -\frac{3}{7}$ and $\cos\theta < 0$

d) $\csc\theta = 3$ and $\tan\theta < 0$

e) $\sec\theta = 2$ and $\csc\theta < 0$

f) $\cot\theta = \frac{2}{3}$ and $\sin\theta < 0$

g) $\sin\theta = 0$ and $\sec\theta = -1$

h) $\cot\theta = -1$ and $\cos\theta > 0$

3. Write the first expression in terms of the second if the terminal point determined by t is in the given quadrant.

a) $\sin\theta$ in terms of $\cos\theta$ if θ in quadrant III.

b) $\sin\theta$ in terms of $\tan\theta$ if θ is in quadrant II.

c) $\cos\theta$ in terms of $\csc\theta$ if θ is in quadrant IV.

d) $\tan\theta$ in terms of $\sin\theta$ if θ is in quadrant IV.

e) $\tan\theta$ in terms of $\csc\theta$ if θ is in quadrant III.

f) $\csc\theta$ in terms of $\cos\theta$ if θ is in quadrant IV.

g) $\csc\theta$ in terms of $\tan\theta$ if θ is in quadrant IV.

h) $\cot\theta$ in terms of $\sec\theta$ if θ is in quadrant III.

i) $\sec\theta$ in terms of $\tan\theta$ if θ is in quadrant II.

4. If $\tan\theta = \frac{-3}{4}$ and θ is in quadrant IV, find $\sin\theta + \cos\theta$.

5. If $\cos\theta = \frac{-3}{4}$ and θ is in quadrant II, find $\cot\theta + \csc\theta$.

6. Find the exact value of each expression. Do not use a calculator.

a) $\sin \frac{\pi}{3} - \cos \frac{\pi}{6}$

b) $\tan \frac{\pi}{7} \cot \frac{\pi}{7}$

c) $\csc \frac{2\pi}{3} - \tan \frac{5\pi}{6}$

d) $\sin \frac{\pi}{4} - \csc \frac{\pi}{4}$

e) $\sin \frac{-2\pi}{3} + \cos \frac{2\pi}{3}$

f) $\cot \frac{2\pi}{5} - \frac{\cos \frac{2\pi}{5}}{\sin \frac{2\pi}{5}}$

g) $\sin^2 17^\circ + \cos^2 17^\circ$

h) $\tan^2 \frac{\pi}{9} - \sec^2 \frac{\pi}{9}$

i) $\cot 3^\circ - \frac{\cos 3^\circ}{\sin 3^\circ}$

j) $\tan \frac{-\pi}{3} + \cot \frac{11\pi}{6}$

7. Find the area of the triangle with sides of length 3cm and 7cm and included angle 56° .

8. Find the area of an equilateral triangle with sides of length 15cm.

9. A triangle has an area of 35cm^2 . If two adjacent sides have length 21cm and 13cm, what is the angle between the two sides?

ANSWERS

1. a) $\frac{\sqrt{3}}{2}$ b) $\frac{-\sqrt{2}}{2}$ c) $-\frac{\sqrt{3}}{2}$ d) 1 e) $\frac{-1}{2}$ f) $\frac{\sqrt{3}}{2}$ g) -1 h) 0
 i) $\frac{\sqrt{2}}{2}$ j) 0 k) $\sqrt{3}$ l) 2 m) $\frac{-2\sqrt{3}}{3}$ n) -1 o) -1 p) $-\sqrt{2}$
 q) -1 r) $\sqrt{2}$ s) -2 t) 0 u) -2 v) $\cancel{\neq}$ w) 2 x) 1
2. a) $\sin \theta = \frac{5}{13}$ $\csc \theta = \frac{13}{5}$ b) $\sin \theta = -\frac{4}{5}$ $\csc \theta = -\frac{5}{4}$
 $\cos \theta = -\frac{12}{13}$ $\sec \theta = -\frac{13}{12}$ $\cos \theta = -\frac{3}{5}$ $\sec \theta = -\frac{5}{3}$
 $\tan \theta = -\frac{5}{12}$ $\cot \theta = -\frac{12}{5}$ $\tan \theta = \frac{4}{3}$ $\cot \theta = \frac{3}{4}$
- c) $\sin \theta = \frac{3\sqrt{58}}{58}$ $\csc \theta = \frac{\sqrt{58}}{3}$ d) $\sin \theta = \frac{1}{3}$ $\csc \theta = 3$
 $\cos \theta = -\frac{7\sqrt{58}}{58}$ $\sec \theta = -\frac{\sqrt{58}}{7}$ $\cos \theta = -\frac{2\sqrt{2}}{3}$ $\sec \theta = -\frac{3\sqrt{2}}{4}$
 $\tan \theta = -\frac{3}{7}$ $\cot \theta = -\frac{7}{3}$ $\tan \theta = -\frac{\sqrt{2}}{4}$ $\cot \theta = -2\sqrt{2}$
- e) $\sin \theta = -\frac{\sqrt{3}}{2}$ $\csc \theta = -\frac{2\sqrt{3}}{3}$ f) $\sin \theta = -\frac{3\sqrt{13}}{13}$ $\csc \theta = -\frac{\sqrt{13}}{3}$
 $\cos \theta = \frac{1}{2}$ $\sec \theta = 2$ $\cos \theta = -\frac{2\sqrt{13}}{13}$ $\sec \theta = -\frac{\sqrt{13}}{2}$
 $\tan \theta = -\sqrt{3}$ $\cot \theta = -\frac{\sqrt{3}}{3}$ $\tan \theta = \frac{3}{2}$ $\cot \theta = \frac{2}{3}$
- g) $\sin \theta = 0$ $\csc \theta = \cancel{\neq}$ f) $\sin \theta = -\frac{\sqrt{2}}{2}$ $\csc \theta = -\sqrt{2}$
 $\cos \theta = -1$ $\sec \theta = -1$ $\cos \theta = \frac{\sqrt{2}}{2}$ $\sec \theta = \sqrt{2}$
 $\tan \theta = 0$ $\cot \theta = \cancel{\neq}$ $\tan \theta = -1$ $\cot \theta = -1$
3. a) $\sin \theta = -\sqrt{1 - \cos^2 \theta}$ b) $\sin \theta = -\frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}$ c) $\cos \theta = -\frac{\sqrt{\csc^2 \theta - 1}}{\csc \theta}$
 d) $\tan \theta = \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$ e) $\tan \theta = \frac{1}{\sqrt{\csc^2 \theta - 1}}$ f) $\csc \theta = -\frac{1}{\sqrt{1 - \cos^2 \theta}}$
 g) $\csc \theta = \frac{\sqrt{1 + \tan^2 \theta}}{\tan \theta}$ h) $\cot \theta = \frac{1}{\sqrt{\sec^2 \theta - 1}}$ i) $\sec \theta = -\sqrt{1 + \tan^2 \theta}$
4. $\frac{1}{5}$
 5. $\frac{\sqrt{7}}{7}$
6. a) 0 b) 1 c) $\sqrt{3}$ d) $\frac{-\sqrt{2}}{2}$ e) $\frac{-1-\sqrt{3}}{2}$ f) 0 g) 1 h) -1
 i) 0 j) $-2\sqrt{3}$
7. 8.7 cm^2
 8. 97.4 cm^2
 9. 14.9° or 165.1°