MATHEMATICS 201-009-50
Precalculus
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XV -Exponential and Logarithmic Equations

1. Find the solution to the following equation, correct to four significant digits.
   a) $3^x = 7$
   b) $8^{-x} = 3$
   c) $2e^x = 5$
   d) $4^{2x} = 56$
   e) $10^{\frac{-2}{3}x} = 7$
   f) $2e^{3-5x} + 3 = 7$
   g) $\left(\frac{1}{5}\right)^x = 3$
   h) $3^x = 7^{x+1}$
   i) $2^{2x-1} = 5^{3x+2}$
   j) $4^{\frac{1}{x}} = 23$
   k) $2 - 3^{4x-7} = \frac{1}{5}$
   l) $\frac{20}{1+e^{-x}} = 5$
   m) $\ln x = 7$
   n) $\ln(2x-1) = \frac{1}{2}$
   o) $\log_4(2x+1) = 5$
   p) $\log_3\sqrt{4x+1} = 2$

2. Solve the following equations.
   a) $\left(\frac{1}{2}\right)^{x-3} = 64$
   b) $3^{2x-1} = 81$
   c) $\ln x - \ln 4 = 0$
   d) $\log_2 x = -3$
   e) $\log x = -1$
   f) $\ln(2x-1) = 4$
   g) $e^{\ln(2x-1)} = 3$
   h) $\ln e^{3x+1} = 16$
   i) $x^2e^{5x} - e^{5x} = 0$
   j) $e^{2x} - 3e^x - 10 = 0$
   k) $e^{2x} - e^x - 6 = 0$
   l) $e^{3x} - 2e^{2x} + e^x = 0$
   m) $-2e^{-x^2} + 4x^2e^{-x^2} = 0$
   n) $x^2e^{2x} + 2xe^{2x} = 8e^{2x}$
   o) $\ln(x^2) = 4$
   p) $4 - \ln(2x+1) = 7$
   q) $\log_2(x^2 - 5x - 2) = 2$
   r) $\log_4 3 + \log_4 x = \log_4 2 + \log_4(x+1)$
   s) $\log x + \log(x-1) = \log 4x$
   t) $\ln 4x - 3\ln x^2 = \ln 2$
   u) $\ln(x-3) + \ln(x+6) = \ln 10$
   v) $\log_8 x - \log_8(x-1) = \frac{1}{3}$
   w) $\log_3(x+2) - \log_3 x = \log_3(x+4)$
   x) $\ln x + \ln(x+1) = 1$
   y) $\ln(x+2) - \ln(x+3) = 4$
   z) $\log_3(\log_4 x) = 2$
   aa) $\log_2 x^{\log_2 x} = 9$
   bb) $\left(\sqrt[3]{2}\right)^{2-x} = 2^{x^2}$

3. The population of a city in 2000 was 100 000, and the estimated relative growth 1.5%.
   a) What will be the population in 2010?
   b) How long will it take for the population to reach 150 000?

4. A culture of bacteria triples in size every 30 minutes. How long does it take for the culture to double in size?

5. The count in a culture of bacteria is 200 after 2 hours and 350 after 3 hours.
   a) What was the initial size of the population?
   b) How long does it take for the population to triple in size?

6. A person carrying a highly infectious virus arrives home after a safari in Africa. For this particular virus, the contagion is such that the number of infected people doubles every 3 hours.
   a) How many people will be infected one day after he has arrived?
   b) How many people will be infected two days after he has arrived?
   c) How long will it take for the whole town to be infected (if the town has 100 000 inhabitants?

7. A man invest $6000 in a mutual fund which pays 10% per year, compounded monthly.
   a) How much money will the man have in 5 years?
   b) How long would it take for the amount to be $20 000?

8. Find the time required for an investment to double if the interest is 8% compounded semi-annually?

9. Find the time required for an investment to triple if the interest is 4.5% compounded continuously.

10. The sales S (in thousands of units) of a new product after it has been on the market for t years are modeled by $S(t) = 100\left(1 - e^{kt}\right)$. Twenty thousand units of the new product were sold during the first year. Use this model to estimate the number of units sold after 8 years.

11. A sky diver jumps off an airplane. The air resistance he experiences is proportional to his velocity. It can be shown that the velocity of the sky diver at time t is then given by $v(t) = 98\left(e^{-0.1t} - 1\right)$ where t is in seconds and v in meters per second. How long will it take for the velocity to reach -50 m/s.

12. One student in a college organization of 200 members proceeds to spread a rumor. The number of students y that know about the rumor after t days is given by $y(t) = \dfrac{200}{1 + Ae^{kt}}$. If three students are informed after one day, then how many students know the rumor after 2 days? When will half of the college have heard about the rumor?

13. According to **Newton's law of Cooling**, the temperature $T(t)$ of a heated object decreases exponentially over time toward the temperature of the surrounding medium. This gives us

$$T(t) = T_s + (T_o - T_s)e^{kt}$$

Where T_s is the temperature of the surrounding medium (such as air temperature) and T_o the initial temperature of the heated object. A hot cup of tea is initially at 100°C when poured. How long does it take for the tea to reach a temperature of 50°C if it is 80°C after 15 minutes and the room temperature is 30°C?

14. Use Newton's law of cooling to determine the time of death if a corpse is at 26°C when discovered at 3:00 PM and 20°C three hours later. Assume that the temperature of the surrounding is 15°C. (Normal body temperature is 37°C.)

ANSWERS

1. a) 1.771 b) -0.5286 c) 0.9163 d) 1.452
   e) -1.268 f) 0.4614 g) -0.6830 h) -2.298
   i) -1.137 j) 0.4422 k) 1.884 l) -1.099
   m) 1097 n) 1.324 o) 511.5 p) 20
2. a) -3 b) $\frac{5}{2}$ c) 4 d) $\frac{1}{8}$
   e) $\frac{1}{10}$ f) $\frac{1}{2}e^4 + \frac{1}{2}$ g) 2 h) 5
   i) ± 1 j) $\ln 5$ k) $\ln 3$ l) 0
   m) $\pm\frac{\sqrt{2}}{2}$ n) -4, 2 o) $\pm e^2$ p) $\frac{1-e^3}{2e^3}$
   q) -1, 6 r) 2 s) 5 t) $\sqrt[5]{2}$
   u) 4 v) 2 w) $\frac{-3}{2} + \frac{1}{2}\sqrt{17}$ x) $\frac{2e}{1+\sqrt{4e+1}}$
   y) $\frac{2-3e^4}{e^4-1}$ z) 262144 aa) $\frac{1}{8}$, 8 bb) -1, $\frac{2}{3}$
3. a) 116 183 b) ≈ 27 years
4. ≈ 18.9 minutes
5. a) 65 b) ≈1.96 hours
6. a) 256 b) 65536 c) ≈ 49.8 hours
7. a) \$9871.85 b) ≈12.1 years
8. ≈8.8 years
9. ≈24.4 years
10. 83223 units
11. ≈ 7.14 seconds
12. 9 students 4.8 days
13. $t \approx 55.85$ min
14. $t \approx -2.64$ hours, 12:21 PM.