MATHEMATICS 201-009-50
Precalculus
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XIX – Inverse Trigonometric Functions

1. Find the exact value for the inverse trigonometric function at the given number (Do not use a calculator!)

a) $\arcsin 1$ b) $\arcsin\left(\frac{-\sqrt{2}}{2}\right)$ c) $\arccos(-1)$
d) $\arctan 1$ e) $\arccos\left(-\frac{1}{2}\right)$ f) $\arctan\sqrt{3}$
g) $\arctan\frac{\sqrt{3}}{3}$ h) $\arcsin\frac{\sqrt{3}}{2}$ i) $\arccos 0$
j) $\arcsin\left(\frac{-1}{2}\right)$ k) $\arccos\frac{\sqrt{3}}{2}$ l) $\arctan 0$
m) $\operatorname{arcsec} 2$ n) $\operatorname{arccot} 0$ o) $\operatorname{arccsc}\left(-\sqrt{2}\right)$
p) $\operatorname{arcsec}\sqrt{2}$ q) $\operatorname{arcsec}\left(-\sqrt{2}\right)$ r) $\operatorname{arccsc}\frac{2\sqrt{3}}{3}$
s) $\operatorname{arcsec} 1$ t) $\operatorname{arccsc}(-2)$ u) $\operatorname{arccot}\left(-\sqrt{3}\right)$

2. Find the exact value of each expression.

a) $\sin\left(\arcsin\frac{2}{5}\right)$ b) $\arccos\left(\cos\frac{-\pi}{3}\right)$ c) $\arcsin\left(\sin\frac{7\pi}{6}\right)$
d) $\arccos\left(\cos\frac{12\pi}{7}\right)$ e) $\arctan\left(\tan\frac{3\pi}{4}\right)$ f) $\tan\left(\arctan(-1)\right)$
g) $\sin\left(\arcsin 5\right)$ h) $\operatorname{arcsec}\left(\sec\frac{\pi}{3}\right)$ i) $\operatorname{arccsc}\left(\csc\frac{\pi}{4}\right)$

3. Find the exact value of each expression.

a) $\sin\left(\arccos\frac{4}{5}\right)$ b) $\cos\left(\arctan\frac{4}{3}\right)$ c) $\tan\left(\arcsin\frac{12}{13}\right)$
d) $\sec\left(\arccos\frac{4}{7}\right)$ e) $\cot\left(\arcsin\frac{1}{5}\right)$ f) $\csc\left(\arctan 3\right)$
g) $\sin\left(\operatorname{arcsec}\frac{13}{5}\right)$ h) $\tan\left(\operatorname{arccsc} 2\right)$ i) $\cos\left(\operatorname{arccot}\frac{3}{4}\right)$
j) $\sec\left(\operatorname{arccsc}\sqrt{2}\right)$ k) $\cot\left(\operatorname{arcsec}\frac{3}{2}\right)$ l) $\sin\left(\arccos\frac{-3}{5}\right)$
m) $\tan\left(\arcsin\frac{-12}{13}\right)$ n) $\cos\left(\arctan(-2)\right)$ o) $\sec\left(\arcsin\frac{-3}{4}\right)$

4. Complete the identities

a) $\sin\left(\arccos x\right)=?$ b) $\cos\left(\arctan x\right)=?$ c) $\cot\left(\operatorname{arccsc} x\right)=?$
d) $\sin\left(\arctan x\right)=?$ e) $\tan\left(\operatorname{arccot} x\right)=?$ f) $\tan\left(\arccos x\right)=?$

5. Prove the following identities

a) $\operatorname{arcsec} x=\arccos\frac{1}{x}$ if $|x|\geq 1$
b) $\operatorname{arccot} x=\arctan\frac{1}{x}$ if $x>0$

ANSWERS

1. a) $\frac{\pi}{2}$ b) $\frac{-\pi}{4}$ c) π d) $\frac{\pi}{4}$ e) $\frac{2\pi}{3}$ f) $\frac{\pi}{3}$
   g) $\frac{\pi}{6}$ h) $\frac{\pi}{3}$ i) $\frac{\pi}{2}$ j) $\frac{-\pi}{6}$ k) $\frac{\pi}{6}$ l) 0
   m) $\frac{\pi}{3}$ n) $\frac{\pi}{2}$ o) $\frac{-\pi}{4}$ p) $\frac{\pi}{4}$ q) $\frac{3\pi}{4}$ r) $\frac{\pi}{3}$
   s) 0 t) $\frac{-\pi}{6}$ u) $\frac{5\pi}{6}$
2. a) $\frac{2}{5}$ b) $\frac{\pi}{3}$ c) $\frac{-\pi}{6}$ d) $\frac{2\pi}{7}$ e) $\frac{-\pi}{4}$ f) -1
   g) $\nexists$ h) $\frac{\pi}{3}$ i) $\frac{\pi}{4}$
3. a) $\frac{3}{5}$ b) $\frac{3}{5}$ c) $\frac{12}{5}$ d) $\frac{7}{4}$ e) $2\sqrt{6}$ f) $\frac{\sqrt{10}}{3}$
   g) $\frac{12}{13}$ h) $\frac{\sqrt{3}}{3}$ i) $\frac{3}{5}$ j) $\sqrt{2}$ k) $\frac{2\sqrt{5}}{5}$ l) $\frac{4}{5}$
   m) $\frac{-12}{5}$ n) $\frac{\sqrt{5}}{5}$ o) $\frac{4\sqrt{7}}{7}$
4. a) $\sin(\arccos x) = \sqrt{1-x^2}$ b) $\cos(\arctan x) = \dfrac{1}{\sqrt{1+x^2}}$ c) $\cot(\operatorname{arccsc} x) = \sqrt{x^2-1}$
   d) $\sin(\arctan x) = \dfrac{x}{\sqrt{1+x^2}}$ e) $\tan(\operatorname{arccot} x) = \dfrac{1}{x}$ f) $\tan(\arccos x) = \dfrac{\sqrt{1-x^2}}{x}$
5. a) Let $\operatorname{arcsec} x = \theta$
   Then $\sec\theta = x$

$$\frac{1}{\cos\theta} = x$$

$$\cos\theta = \frac{1}{x}$$

$$\theta = \arccos\left(\tfrac{1}{x}\right)$$

   Thus $\operatorname{arcsec} x = \arccos\frac{1}{x}$

   b) Let $\operatorname{arccot} x = \theta$
   Then $\cot\theta = x$

$$\frac{1}{\tan\theta} = x$$

$$\tan\theta = \frac{1}{x}$$

$$\theta = \arctan\left(\tfrac{1}{x}\right)$$

   Thus $\operatorname{arccot} x = \arctan\frac{1}{x}$