MATHEMATICS 201-009-50
Precalculus
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XIV – Logarithmic Functions

1. Sketch the graph of the following function. State the domain, range, intercepts and asymptotes.
   a) $f(x)=\log_3(x+1)$
   b) $f(x)=\log_2 x+1$
   c) $f(x)=-\log x$
   d) $f(x)=-\log_4(-x)$
   e) $f(x)=\log_2(x-1)+3$
   f) $f(x)=\ln(x-2)$
   g) $f(x)=\ln(2-x)$
   h) $f(x)=\ln|x|$

2. Express the equation in exponential form.
   a) $\log_8 512=3$
   b) $\log_{27} 9=\frac{2}{3}$
   c) $\log\frac{1}{100}=-2$
   d) $\ln 1=0$
   e) $\log_2 x=5$
   f) $\ln x=3$
   g) $\ln(x-2)=4$
   h) $\ln 2=x$

3. Express the equation in logarithmic form.
   a) $4^{\frac{-3}{2}}=\frac{1}{8}$
   b) $81^{\frac{1}{2}}=9$
   c) $10^3=1000$
   d) $\left(\frac{1}{2}\right)^{-4}=16$
   e) $4^x=5$
   f) $e^{\frac{1}{2}t}=3$
   g) $e^{2x-1}=3$
   h) $e^3=x$

4. Evaluate exactly (*without the use of a calculator*).
   a) $\log_2 2^3$
   b) $\log_3 81$
   c) $\log_5 125$
   d) $\log_4 1$
   e) $\log_9 3$
   f) $\log_{\frac{1}{4}} 16$
   g) $\log_2 8^{33}$
   h) $\log_9 \sqrt{3}$
   i) $\log\sqrt{10}$
   j) $e^{\ln\pi}$
   k) $\ln\frac{1}{e}$
   l) $10^{\log 12}$
   m) $\log_8 0.25$
   n) $\log_3\sqrt{27}$
   o) $\log_{12} 9+\log_{12} 16$
   p) $\log 5+\log 200$
   q) $\log_4 192-\log_4 3$
   r) $10^{3\log 5}$
   s) $\log 2^{\log 1}$
   t) $e^{2\ln 7}$

5. Rewrite each expression in a form with no logarithms of products, quotients, or powers.

a) $\log_4 x(x+2)$

b) $\log_2\left(\frac{2}{x}\right)$

c) $\log_2\left(\frac{x^2y^3}{z^5}\right)$

d) $\log_6 x\sqrt{y}$

e) $\ln\frac{3x^2}{(x+1)^{10}}$

f) $\log_4\frac{16x^2(x-1)^3}{(x+2)^4}$

g) $\log\sqrt{\frac{x+5}{x-5}}$

h) $\ln\left(x\sqrt{\frac{y}{z}}\right)$

i) $\log_3 x\sqrt[3]{x^2+1}$

j) $\ln\frac{(x+3)^5\sqrt[4]{2x-1}}{2x^{12}}$

k) $\ln\sqrt[3]{\frac{x^2+1}{(x^3+1)^5}}$

l) $\log_2\frac{4^x(x+1)^2}{8\sqrt{x}}$

m) $\log\frac{10\sqrt[3]{x^2+y^2}}{x^2y^4}$

n) $\ln\sqrt{x\sqrt{y\sqrt{z}}}$

6. Rewrite the expression as a single logarithm.

a) $\ln x+\ln 5$

b) $-3\log_4(x+2)$

c) $\log_2 x+\log_2 y^3-2\log_2 z$

d) $2\log(x-1)-\log(x^2-1)$

e) $\ln(x+y)+\ln(x-y)-3\ln x$

f) $\ln 5+2\ln x-3\ln x^5+\frac{1}{2}\ln(x+1)$

g) $4\log x-\frac{1}{3}\log(x^2+1)+2\log(x-1)$

h) $2\log x^3-3\log\sqrt{x}+\frac{1}{2}\log x^5+3\log 2$

i) $\frac{1}{2}\left[\ln x-3\ln(x^2-1)+2\ln(x^2+1)\right]$

j) $2(\ln x-\ln(x-1)-\ln(x+1))$

7. Approximate the value of the logarithm to 4 significant digits.

a) $\log_4 5$

b) $2\log_3\sqrt{7}$

8. Simplify the following

a) $(\log_7 11)(\log_{11} 5)$

b) $\frac{\log_2 24}{\log_2 3}$

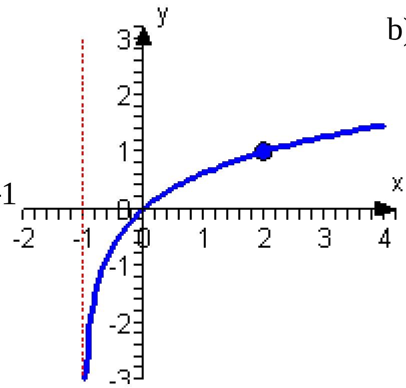
9. Show that $\log_a\left(x+\sqrt{x^2-1}\right)+\log_a\left(x-\sqrt{x^2-1}\right)=0$.

10. Show that $\ln\left(1+e^{2x}\right)=2x+\ln\left(1+e^{-2x}\right)$.

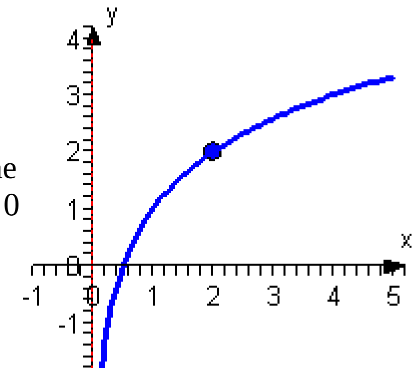
11. Show that $\log_{\frac{1}{a}} x=-\log_a x$.

ANSWERS

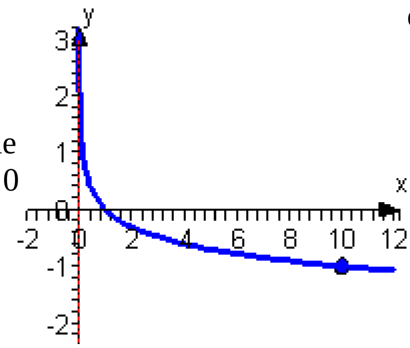
1. a) D: $(-1,\infty)$
R: $\mathbb{R}$
x-ints: 0
y-int: 0
V.A.: $x=-1$



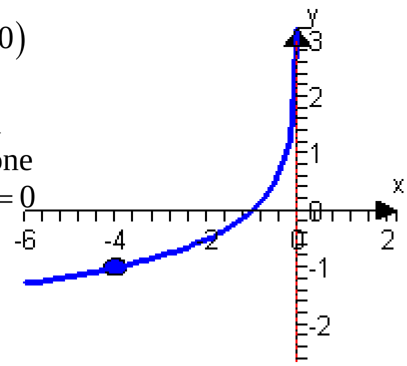
b) D: $(0,\infty)$
R: $\mathbb{R}$
x-ints: $\frac{1}{2}$
y-int: None
V.A.: $x=0$



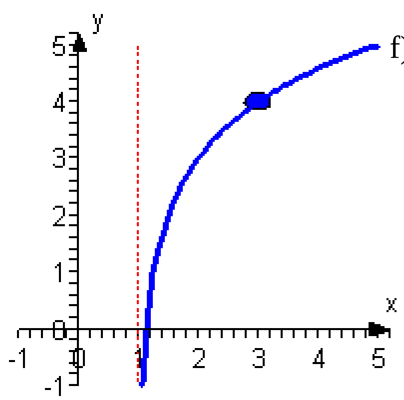
c) D: $(0,\infty)$
R: $\mathbb{R}$
x-ints: 1
y-int: None
V.A.: $x=0$



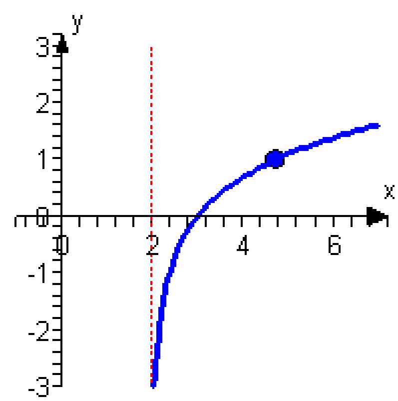
d) D: $(-\infty,0)$
R: $\mathbb{R}$
x-ints: -1
y-int: None
V.A.: $x=0$



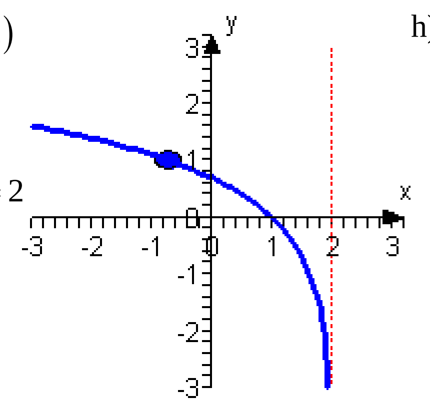
e) D: $(1,\infty)$
R: $\mathbb{R}$
x-ints: $\frac{9}{8}$
y-int: None
V.A.: $x=1$



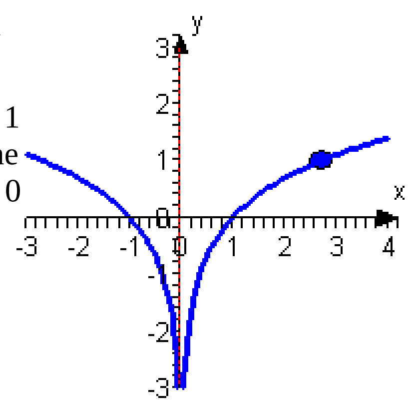
f) D: $(2,\infty)$
R: $\mathbb{R}$
x-ints: 3
y-int: None
V.A.: $x=2$



g) D: $(-\infty,2)$
R: $\mathbb{R}$
x-ints: 1
y-int: 1
V.A.: $x=2$



h) D: $\mathbb{R}/\{0\}$
R: $\mathbb{R}$
x-ints: -1, 1
y-int: None
V.A.: $x=0$



2. a) $8^3 = 512$ b) $27^{\frac{2}{3}} = 9$ c) $10^{-2} = \frac{1}{100}$ d) $e^0 = 1$
   e) $2^5 = x$ f) $e^3 = x$ g) $e^4 = x - 2$ h) $e^x = 2$
3. a) $\log_4 \frac{1}{8} = \frac{-3}{2}$ b) $\log_{81} 9 = \frac{1}{2}$ c) $\log 1000 = 3$ d) $\log_{\frac{1}{2}} 16 = -4$
   e) $\log_4 5 = x$ f) $\ln 3 = \frac{1}{2}t$ g) $\ln 3 = 2x - 1$ i) $\ln x = 3$
4. a) 3 b) 4 c) 3 d) 0
   e) $\frac{1}{2}$ f) -2 g) 99 h) $\frac{1}{4}$
   i) $\frac{1}{2}$ j) π k) -1 l) 12
   m) $\frac{-2}{3}$ n) $\frac{3}{2}$ o) 2 p) 3
   q) 3 r) 125 s) 0 t) 49
5. a) $\log_4 x + \log_4 (x+2)$ b) $1 - \log_2 x$
   c) $2\log_2 x + 3\log_2 y - 5\log_2 z$ d) $\log_6 x + \frac{1}{2}\log_6 y$
   e) $\ln 3 + 2\ln x - 10\ln(x+1)$ f) $2 + 2\log_4 x + 3\log_4(x-1) - 4\log_4(x+2)$
   g) $\frac{1}{2}\log(x+5) - \frac{1}{2}\log(x-5)$ h) $\ln x + \frac{1}{2}\ln y - \frac{1}{2}\ln z$
   i) $\log_3 x + \frac{1}{3}\log_3(x^2+1)$ j) $5\ln(x+3) + \frac{1}{4}\ln(2x-1) - \ln 2 - 12\ln x$
   k) $\frac{1}{3}\ln(x^2+1) - \frac{5}{3}\ln(x^3+1)$ l) $2x + 2\log_2(x+1) - 3 - \frac{1}{2}\log_2 x$
   m) $1 + \frac{1}{3}\log(x^2+y^2) - 2\log x - 4\log y$ n) $\frac{1}{2}\ln x + \frac{1}{4}\ln y + \frac{1}{8}\ln z$
6. a) $\ln(5x)$ b) $\log_4 \frac{1}{(x+2)^3}$ c) $\log_2\left(\frac{xy^3}{z^2}\right)$ d) $\log\frac{x-1}{x+1}$
   e) $\ln\frac{x^2-y^2}{x^3}$ f) $\ln\frac{5\sqrt{x+1}}{x^{13}}$ g) $\log\frac{x^4(x-1)^2}{\sqrt[3]{x^2+1}}$ h) $\log(8x^7)$
   i) $\ln\sqrt{\frac{x(x^2+1)^2}{(x^2-1)^3}}$ j) $\ln\frac{x^2}{(x^2-1)^2}$
7. a) 1.161 b) 1.771
8. a) $\log_7 5$ b) $\log_3 24$
9. $$\log_a\left(x+\sqrt{x^2-1}\right) + \log_a\left(x-\sqrt{x^2-1}\right) = \log_a\left[\left(x+\sqrt{x^2-1}\right)\left(x-\sqrt{x^2-1}\right)\right]$$
$$= \log_a\left(x^2 - (x^2-1)\right) = \log_a 1 = 0$$
10. $$2x + \ln\left(1+e^{-2x}\right) = 2x + \ln\left(1+\frac{1}{e^{2x}}\right) = 2x + \ln\left(\frac{1+e^{2x}}{e^{2x}}\right) = 2x + \ln\left(1+e^{2x}\right) - \ln e^{2x}$$
$$= 2x + \ln\left(1+e^{2x}\right) - 2x = \ln\left(1+e^{2x}\right)$$
11. $\log_{\frac{1}{a}} x = y$
$$\left(\tfrac{1}{a}\right)^y = x$$
$$a^{-y} = x$$
$$-y = \log_a x$$
$$y = -\log_a x$$