

MATHEMATICS 201-009-50

Precalculus

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Fall 2007

XI – Inverse Functions1. For each of the function $f(x)$,i) Find the inverse of $f(x)$ ii) Find the domain and range of $f(x)$ and $f^{-1}(x)$.iii) Verify that $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$.iv) Sketch the graph of $f(x)$ and $f^{-1}(x)$.

a) $f(x) = 2x - 5$

b) $f(x) = x^2 + 3 \quad x \geq 0$

c) $f(x) = \sqrt{x-4}$

d) $f(x) = (x-2)^2 + 3 \quad x \geq 2$

e) $f(x) = \sqrt{x+1} - 3$

f) $f(x) = x^2 - 2x + 3 \quad x \geq 1$

2. Find the inverse of $f(x)$.

a) $f(x) = \frac{1}{x+3}$

b) $f(x) = \frac{2x-1}{x+3}$

c) $f(x) = \frac{3x-7}{x+1}$

d) $f(x) = 4 - x^3$

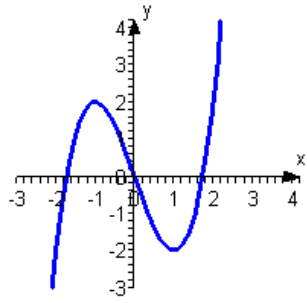
e) $f(x) = \sqrt{9-x^2} \quad 0 \leq x \leq 3$

f) $f(x) = (8-x^3)^5$

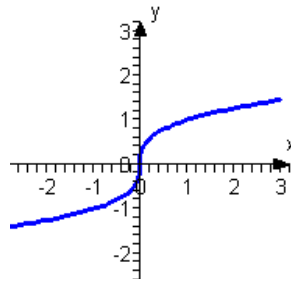
3. Assuming that f is invertible, find $f^{-1}(5)$ if $f(17) = 5$.4. Assuming that f is invertible, find $f(-2)$ if $f^{-1}(\sqrt{3}) = -2$.5. Let $f(x) = 2x^3 + 5x + 3$. Find x if $f^{-1}(x) = 1$.6. Let $f(x) = \frac{x^3}{x^2+1}$. Find x if $f^{-1}(x) = 2$.7. Let $f(x) = \sqrt{4x+1} - 12$. Find $f^{-1}(4)$.

8. Determine whether each the function f whose graph is given is one-to-one.

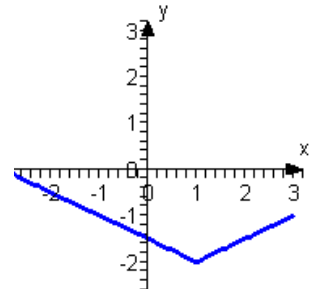
a)



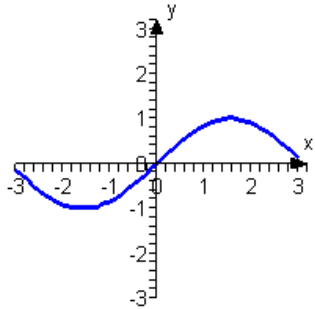
b)



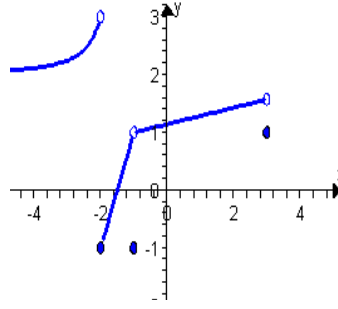
c)



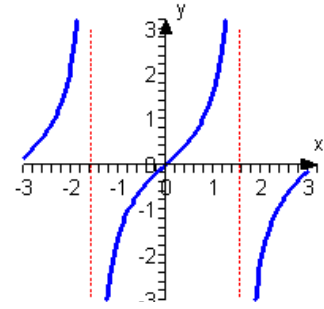
d)



e)



f)

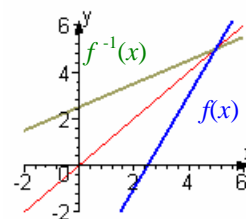


ANSWERS

1. a) i) $f^{-1}(x) = \frac{1}{2}x + \frac{5}{2}$

ii) Domain of $f(x)$: \mathbb{R} Range of $f(x)$: \mathbb{R}

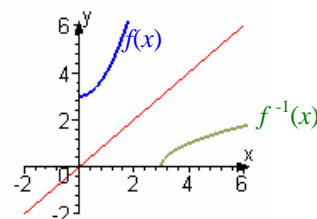
Domain of $f^{-1}(x)$: \mathbb{R} Range of $f^{-1}(x)$: \mathbb{R}



b) i) $f^{-1}(x) = \sqrt{x-3}$

ii) Domain of $f(x)$: $[0, \infty)$ Range of $f(x)$: $[3, \infty)$

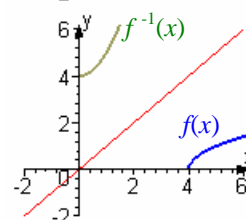
Domain of $f^{-1}(x)$: $[3, \infty)$ Range of $f^{-1}(x)$: $[0, \infty)$



c) i) $f^{-1}(x) = x^2 + 4$

ii) Domain of $f(x)$: $[4, \infty)$ Range of $f(x)$: $[0, \infty)$

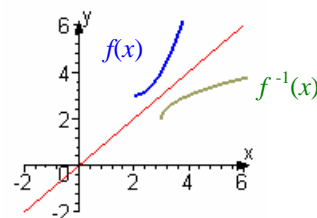
Domain of $f^{-1}(x)$: $[0, \infty)$ Range of $f^{-1}(x)$: $[4, \infty)$



d) i) $f^{-1}(x) = \sqrt{x-3} + 2$

ii) Domain of $f(x)$: $[2, \infty)$ Range of $f(x)$: $[3, \infty)$

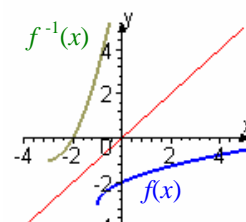
Domain of $f^{-1}(x)$: $[3, \infty)$ Range of $f^{-1}(x)$: $[2, \infty)$



e) i) $f^{-1}(x) = (x+3)^2 - 1$

ii) Domain of $f(x)$: $[-1, \infty)$ Range of $f(x)$: $[-3, \infty)$

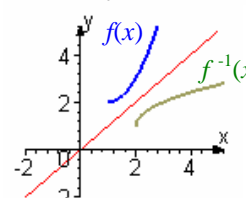
Domain of $f^{-1}(x)$: $[-3, \infty)$ Range of $f^{-1}(x)$: $[-1, \infty)$



f) i) $f^{-1}(x) = \sqrt{x-2} + 1$

ii) Domain of $f(x)$: $[1, \infty)$ Range of $f(x)$: $[2, \infty)$

Domain of $f^{-1}(x)$: $[2, \infty)$ Range of $f^{-1}(x)$: $[1, \infty)$



2. a) $f^{-1}(x) = \frac{1-3x}{x}$

b) $f^{-1}(x) = \frac{3x+1}{2-x}$

c) $f^{-1}(x) = \frac{x+7}{3-x}$

d) $f^{-1}(x) = \sqrt[3]{4-x}$

e) $f^{-1}(x) = \sqrt{9-x^2}$

f) $f^{-1}(x) = \sqrt[3]{8-\sqrt{x}}$

3. 17

4. $\sqrt{3}$

5. 10

6. $\frac{8}{5}$

7. $\frac{255}{4}$

8. Only (b) and (e) are one-to-one.