

MATHEMATICS 201-009-50

Precalculus

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X – Combining Functions

1. For each of the given functions f and g , find the functions

(i) $f + g$

(ii) $f - g$

(iii) fg

(iv) $\frac{f}{g}$

and state the domain of each.

a) $f(x) = 2x + 5$, $g(x) = 3x + 1$

b) $f(x) = 3x^2$, $g(x) = x - 2$

c) $f(x) = \sqrt{1+x}$, $g(x) = \sqrt{1-x}$

d) $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$

e) $f(x) = \frac{1}{x+1}$, $g(x) = \frac{1}{x+2}$

f) $f(x) = \sqrt{x^2 - 1}$, $g(x) = \frac{x^2}{x^2 - 1}$

2. For each of the given functions f and g , find

(i) $(f \circ g)(2)$

(ii) $(g \circ f)(-1)$

(iii) $(f \circ f)(0)$

(iv) $(g \circ g)(3)$

a) $f(x) = 2x + 5$, $g(x) = 3x + 1$

b) $f(x) = 3x^2$, $g(x) = x - 2$

c) $f(x) = \sqrt{1+x}$, $g(x) = \sqrt{1-x}$

d) $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$

e) $f(x) = \frac{1}{x+1}$, $g(x) = \frac{1}{x+2}$

f) $f(x) = \sqrt{x^2 - 1}$, $g(x) = \frac{x^2}{x^2 - 1}$

3. For each of the given functions f and g , find the functions

(i) $f \circ g$

(ii) $g \circ f$

(iii) $f \circ f$

(iv) $g \circ g$

and state the domain of each.

a) $f(x) = 2x + 5$, $g(x) = 3x + 1$

b) $f(x) = 3x^2$, $g(x) = x - 2$

c) $f(x) = \sqrt{1+x}$, $g(x) = \sqrt{1-x}$

d) $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$

e) $f(x) = \frac{1}{x+1}$, $g(x) = \frac{1}{x+2}$

g) $f(x) = x^{\frac{2}{3}}$, $g(x) = x^6$

i) $f(x) = \frac{x+1}{x+2}$, $g(x) = \frac{3}{x+4}$

f) $f(x) = \sqrt{x^2-1}$, $g(x) = \frac{x^2}{x^2-1}$

h) $f(x) = x^2+1$, $g(x) = \sqrt{x-1}$

j) $f(x) = \sqrt[3]{x+2}+1$, $g(x) = (x-1)^3-2$

4. Find $f \circ g \circ h$

a) $f(x) = x^2+1$, $g(x) = \sqrt{x}$, $h(x) = x+3$

b) $f(x) = x^2-3x+1$, $g(x) = 2x-3$, $h(x) = x+2$

ANSWERS

1. a) $(f+g)(x) = 5x+6$ Domain: \mathbb{R}

$(fg)(x) = 6x^2+17x+5$ Domain: \mathbb{R}

b) $(f+g)(x) = 3x^2+x-2$ Domain: \mathbb{R}

$(fg)(x) = 3x^3-6x^2$ Domain: \mathbb{R}

c) $(f+g)(x) = \sqrt{1+x} + \sqrt{1-x}$ Domain: $[-1,1]$

$(f-g)(x) = \sqrt{1+x} - \sqrt{1-x}$ Domain: $[-1,1]$

$(fg)(x) = \sqrt{1-x^2}$ Domain: $[-1,1]$

d) $(f+g)(x) = \frac{x+1}{x^2}$ Domain: $\mathbb{R}/\{0\}$

$(fg)(x) = \frac{1}{x^3}$ Domain: $\mathbb{R}/\{0\}$

e) $(f+g)(x) = \frac{2x+3}{(x+1)(x+2)}$ Domain: $\mathbb{R}/\{-2,-1\}$

$(fg)(x) = \frac{1}{(x+1)(x+2)}$ Domain: $\mathbb{R}/\{-2,-1\}$

f) $(f+g)(x) = \sqrt{x^2-1} + \frac{x^2}{x^2-1}$ Domain: $(-1,1)$

$(fg)(x) = \frac{x^2}{\sqrt{x^2-1}}$ Domain: $(-1,1)$

$(f-g)(x) = -x+4$ Domain: \mathbb{R}

$(\frac{f}{g})(x) = \frac{2x+5}{3x+1}$ Domain: $\mathbb{R}/\{-\frac{1}{3}\}$

$(f-g)(x) = 3x^2-x+2$ Domain: \mathbb{R}

$(\frac{f}{g})(x) = \frac{3x^2}{x-2}$ Domain: $\mathbb{R}/\{2\}$

$(\frac{f}{g})(x) = \sqrt{\frac{1+x}{1-x}}$ Domain: $[-1,1)$

$(f-g)(x) = \frac{x-1}{x^2}$ Domain: $\mathbb{R}/\{0\}$

$(\frac{f}{g})(x) = x$ Domain: $\mathbb{R}/\{0\}$

$(f-g)(x) = \frac{1}{(x+1)(x+2)}$ Domain: $\mathbb{R}/\{-2,-1\}$

$(\frac{f}{g})(x) = \frac{x+2}{x+1}$ Domain: $\mathbb{R}/\{-2,-1\}$

$(f-g)(x) = \sqrt{x^2-1} - \frac{x^2}{x^2-1}$ Domain: $(-1,1)$

$(\frac{f}{g})(x) = \frac{(x^2-1)^{\frac{3}{2}}}{x^2}$ Domain: $(-1,1)$

2. a) $(f \circ g)(2) = 19$ $(g \circ f)(-1) = 10$ $(f \circ f)(0) = 15$ $(g \circ g)(3) = 31$
 b) $(f \circ g)(2) = 0$ $(g \circ f)(-1) = 1$ $(f \circ f)(0) = 0$ $(g \circ g)(3) = -1$
 c) $(f \circ g)(2) \nexists$ $(g \circ f)(-1) = 1$ $(f \circ f)(0) = \sqrt{2}$ $(g \circ g)(3) = \nexists$
 d) $(f \circ g)(2) = 4$ $(g \circ f)(-1) = 1$ $(f \circ f)(0) = \nexists$ $(g \circ g)(3) = 81$
 e) $(f \circ g)(2) = \frac{4}{5}$ $(g \circ f)(-1) \nexists$ $(f \circ f)(0) = \frac{1}{2}$ $(g \circ g)(3) = \frac{5}{11}$
 f) $(f \circ g)(2) = \frac{\sqrt{7}}{3}$ $(g \circ f)(-1) = 0$ $(f \circ f)(0) \nexists$ $(g \circ g)(3) = \frac{81}{17}$
3. a) $(f \circ g)(x) = 6x + 7$ Domain : \mathbb{R} $(g \circ f)(x) = 6x + 16$ Domain : \mathbb{R}
 $(f \circ f)(x) = 4x + 15$ Domain : \mathbb{R} $(g \circ g)(x) = 9x + 4$ Domain : \mathbb{R}
 b) $(f \circ g)(x) = 3(x-2)^2$ Domain : \mathbb{R} $(g \circ f)(x) = 3x^2 - 2$ Domain : \mathbb{R}
 $(f \circ f)(x) = 27x^4$ Domain : \mathbb{R} $(g \circ g)(x) = x - 4$ Domain : \mathbb{R}
 c) $(f \circ g)(x) = \sqrt{1 + \sqrt{1-x}}$ Domain : $(-\infty, 1]$ $(g \circ f)(x) = \sqrt{1 - \sqrt{x+1}}$ Domain : $[-1, 0]$
 $(f \circ f)(x) = \sqrt{1 + \sqrt{1+x}}$ Domain : $[-1, \infty)$ $(g \circ g)(x) = \sqrt{1 - \sqrt{1-x}}$ Domain : $[0, 1]$
 d) $(f \circ g)(x) = x^2$ Domain : $\mathbb{R} / \{0\}$ $(g \circ f)(x) = x^2$ Domain : $\mathbb{R} / \{0\}$
 $(f \circ f)(x) = x$ Domain : $\mathbb{R} / \{0\}$ $(g \circ g)(x) = x^4$ Domain : $\mathbb{R} / \{0\}$
 e) $(f \circ g)(x) = \frac{x+2}{x+3}$ Domain : $\mathbb{R} / \{-3, -2\}$ $(g \circ f)(x) = \frac{x+1}{2x+3}$ Domain : $\mathbb{R} / \{\frac{-3}{2}, -1\}$
 $(f \circ f)(x) = \frac{x+1}{x+2}$ Domain : $\mathbb{R} / \{-2, -1\}$ $(g \circ g)(x) = \frac{x+2}{2x+5}$ Domain : $\mathbb{R} / \{\frac{-5}{2}, -2\}$
 f) $(f \circ g)(x) = \frac{\sqrt{2x^2-1}}{x^2-1}$ Domain : $(-\infty, -1) \cup (-1, -\frac{\sqrt{2}}{2}] \cup [\frac{\sqrt{2}}{2}, 1) \cup (1, \infty)$
 $(g \circ f)(x) = \frac{x^2-1}{x^2-2}$ Domain : $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, -1] \cup [1, \sqrt{2}) \cup (\sqrt{2}, \infty)$
 $(f \circ f)(x) = \sqrt{x^2-2}$ Domain : $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$
 $(g \circ g)(x) = \frac{x^4}{2x^2-1}$ Domain : $\mathbb{R} / \{\pm \frac{\sqrt{2}}{2}, \pm 1\}$
 g) $(f \circ g)(x) = x^4$ Domain : \mathbb{R} $(g \circ f)(x) = x^4$ Domain : \mathbb{R}
 $(f \circ f)(x) = x^{\frac{4}{9}}$ Domain : \mathbb{R} $(g \circ g)(x) = x^{36}$ Domain : \mathbb{R}
 h) $(f \circ g)(x) = x$ Domain : $[1, \infty)$ $(g \circ f)(x) = |x|$ Domain : \mathbb{R}
 $(f \circ f)(x) = x^4 + 2x^2 + 2$ Domain : \mathbb{R} $(g \circ g)(x) = \sqrt{\sqrt{x-1}-1}$ Domain : $[2, \infty)$
 i) $(f \circ g)(x) = \frac{x+7}{2x+11}$ Domain : $\mathbb{R} / \{\frac{-11}{2}, -4\}$ $(g \circ f)(x) = \frac{3(x+2)}{5x+9}$ Domain : $\mathbb{R} / \{-2, \frac{-9}{5}\}$
 $(f \circ f)(x) = \frac{2x+3}{3x+5}$ Domain : $\mathbb{R} / \{-2, \frac{-5}{3}\}$ $(g \circ g)(x) = \frac{3(x+4)}{4x+19}$ Domain : $\mathbb{R} / \{-4, \frac{-19}{4}\}$
 j) $(f \circ g)(x) = x$ Domain : \mathbb{R} $(g \circ f)(x) = x$ Domain : \mathbb{R}
 $(f \circ f)(x) = \sqrt[3]{\sqrt[3]{x+2}+1}+1$ Domain : \mathbb{R} $(g \circ g)(x) = ((x-1)^3 - 3)^3 - 2$ Domain : \mathbb{R}
4. a) $(f \circ g \circ h)(x) = x + 4$ b) $(f \circ g \circ h)(x) = 4x^2 - 2x - 1$