MATHEMATICS 201-009-50
Precalculus
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VII - Functions

1. Let $f(x)=3x-5$. Find
   a) $f(4)$ b) $f(-2)$ c) $f(a)$ d) $f(a+3)$
   e) $f(x)+f(h)$ f) $f(x+h)$ g) $\dfrac{f(x+h)-f(x)}{h}$

2. Let $f(x)=2x^2-4x+2$. Find
   a) $f(-1)$ b) $f\left(\sqrt{2}\right)$ c) $f\left(\sqrt{2}+3\right)$ d) $f\left(t^2\right)$
   e) $f(2x)$ f) $f(3x-4)$ g) $\dfrac{f(x+h)-f(x)}{h}$

3. Let $f(x)=x^3-2x+4$. Find
   a) $f(2)$ b) $f(0)$ c) $f(\pi)$ d) $f\left(x^2\right)$
   e) $\left[f(x)\right]^2$ f) $f(-x)$ g) $\dfrac{f(x+h)-f(x)}{h}$

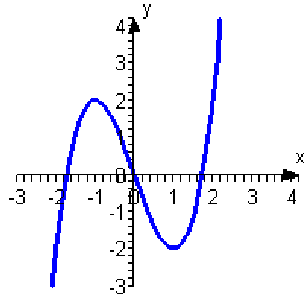
4. Let $f(x)=\dfrac{2-x}{2+x}$. Find
   a) $f(5)$ b) $f(-2)$ c) $f\left(\frac{1}{2}\right)$ d) $f(x+3)$
   e) $f\left(\frac{1}{x}\right)$ f) $f\left(\frac{x}{x-1}\right)$ g) $\dfrac{f(x+h)-f(x)}{h}$

5. Let $f(x)=\sqrt{x^2-5}$. Find
   a) $f(3)$ b) $f\left(\sqrt{30}\right)$ c) $f\left(\sqrt{x}+1\right)$ d) $f\left(\sqrt{x^2+5}\right)$

6. Find the domain of the functions.
   a) $f(x)=\dfrac{1}{x^2-4}$ b) $f(x)=\dfrac{x+2}{x^2-7x+12}$ c) $f(x)=\dfrac{x+3}{x^2+9}$
   d) $f(x)=\dfrac{2x-6}{x^2-2x-3}$ e) $f(x)=\dfrac{x^3+1}{x^3-x}$ f) $f(x)=\sqrt{x+2}$
   g) $f(x)=2-\sqrt{x-4}$ h) $f(x)=\dfrac{\sqrt{x+5}}{x-1}$ i) $f(x)=\dfrac{2x-1}{\sqrt{4x+3}}$
   j) $f(x)=\sqrt[3]{x+1}$ k) $f(x)=\sqrt{x}+\sqrt{2-x}$ l) $f(x)=\sqrt{x^2-x-2}$

m) $f(x)=\sqrt{-x^2+3x+10}$ n) $f(x)=\sqrt{\dfrac{x+2}{x-5}}$

7. Determine whether each curve is the graph of a function of *x*.

a)


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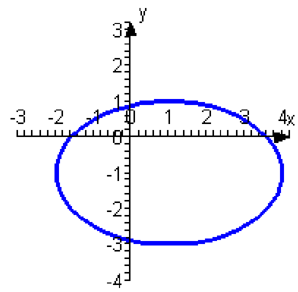

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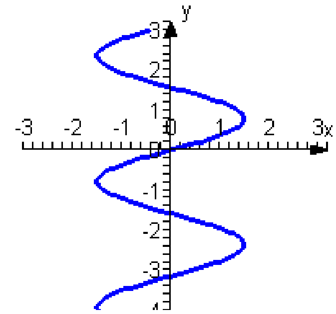

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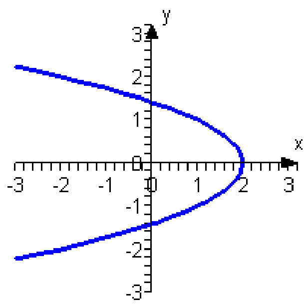

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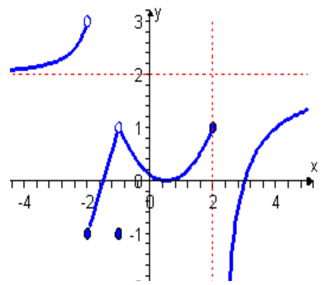

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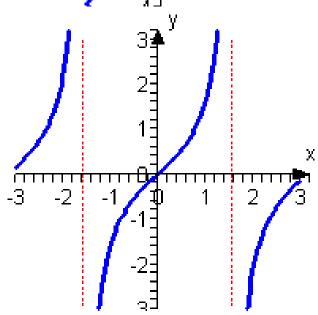

8. Sketch the graph of the following functions.

a) $f(x)=2$ b) $f(x)=2x-3$ c) $f(x)=-x+2$

d) $f(x)=-x^2$ e) $f(x)=x^2+2x+1$ f) $f(x)=1-x^3$

g) $f(x)=\sqrt{-x}$ h) $f(x)=|x+1|$ i) $f(x)=\begin{cases} 2 & x\le 1 \\ -1 & x>1 \end{cases}$

j) $f(x)=\begin{cases} x+1 & x<0 \\ \sqrt{x} & x\ge 0 \end{cases}$ k) $f(x)=\begin{cases} 2x+3 & x\le -1 \\ x-1 & x>-1 \end{cases}$ l) $f(x)=\begin{cases} x-1 & x\ne 2 \\ 3 & x=2 \end{cases}$

m) $f(x)=\begin{cases} -x-2 & x<-1 \\ 2 & -1\le x<2 \\ x & x\ge 2 \end{cases}$ n) $f(x)=\begin{cases} x^2 & |x|<1 \\ 3 & |x|\ge 1 \end{cases}$

9. Use the given graph of *f* to find the domain, the range and to determine the intervals on which *f* is increasing and on which *f* is decreasing.

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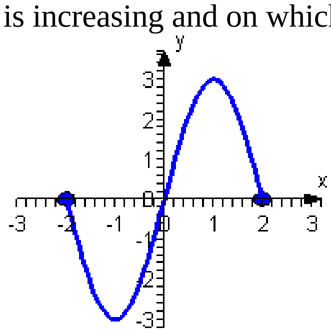

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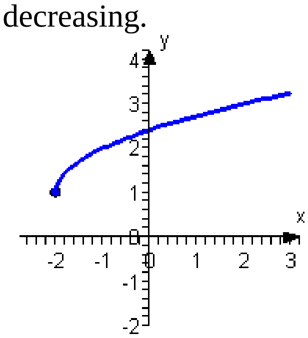

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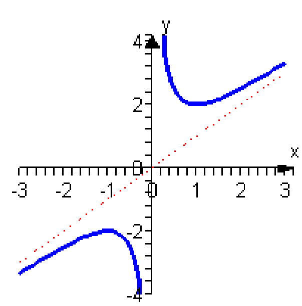

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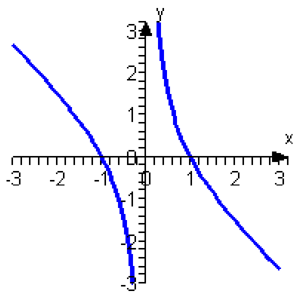

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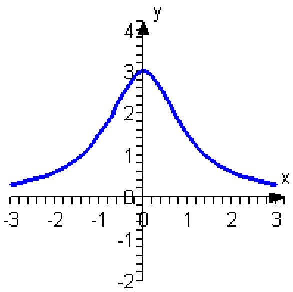

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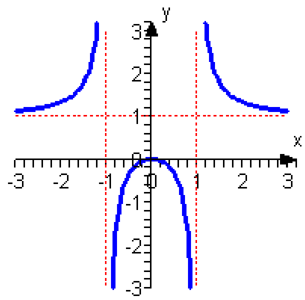
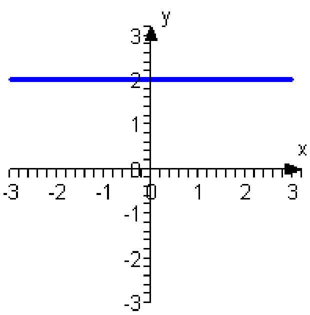


ANSWERS

1. a) 7 b) -11 c) $3a-5$ d) $3a+4$
   e) $3x+3h-10$ f) $3x+3h-5$ g) 3
2. a) 8 b) $6-4\sqrt{2}$ c) $12+8\sqrt{2}$ d) $2t^4-4t^2+2$
   e) $8x^2-8x+2$ f) $18x^2-60x+50$ g) $4x+2h-4$
3. a) 8 b) 4 c) $\pi^3-2\pi+4$ d) x^6-2x^2+4
   e) $x^6-4x^4+8x^3+4x^2-16x+16$ f) $-x^3+2x+4$ g) $3x^2+3xh+h^2-2$
4. a) $\frac{-3}{7}$ b) $\nexists$ c) $\frac{3}{5}$ d) $-\frac{x+1}{x+5}$
   e) $\frac{2x-1}{2x+1}$ f) $\frac{x-2}{3x-2}$ g) $\frac{-4}{(2+x)(2+x+h)}$
5. a) 2 b) 5 c) $\sqrt{x+2\sqrt{x}-4}$ d) $|x|$
6. a) $\mathbb{R}/\{\pm 2\}$ b) $\mathbb{R}/\{-4,-3\}$ c) $\mathbb{R}$ d) $\mathbb{R}/\{-1,3\}$
   e) $\mathbb{R}/\{-1,0,1\}$ f) $[-2,\infty)$ g) $[4,\infty)$ h) $[-5,1)\cup(1,\infty)$
   i) $(-\frac{3}{4},\infty)$ j) $\mathbb{R}$ k) $[0,2]$ l) $(-\infty,-1]\cup[2,\infty)$
   m) $[-2,5]$ n) $(-\infty,-2]\cup(5,\infty)$
7. a) Y b) N c) N d) N e) Y f) Y
8. a)



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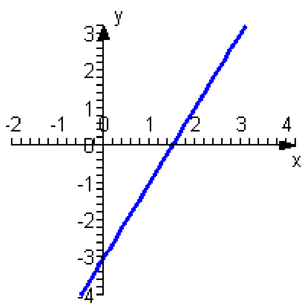

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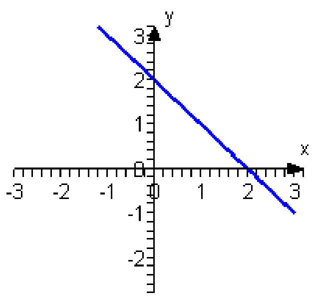

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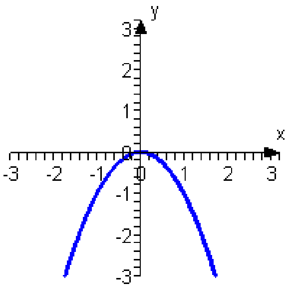

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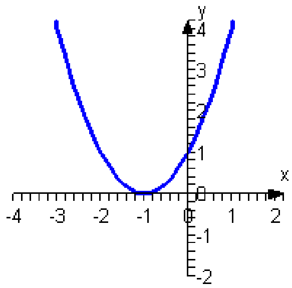

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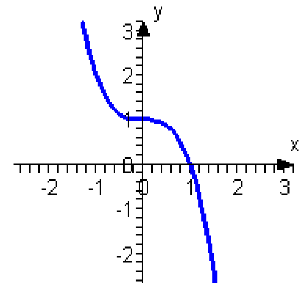

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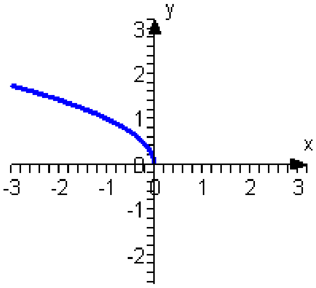

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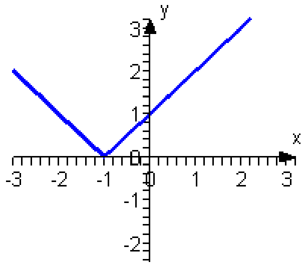

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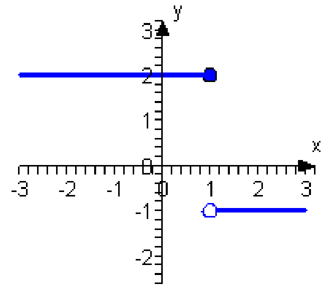

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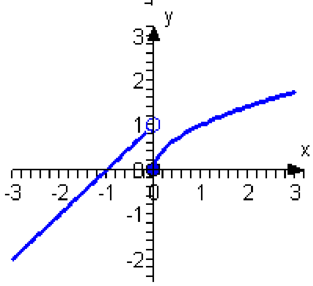

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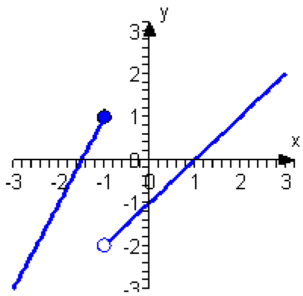

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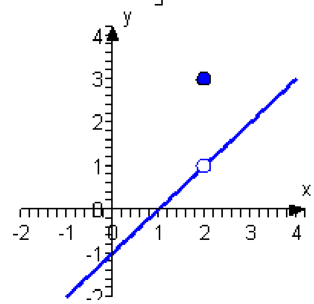

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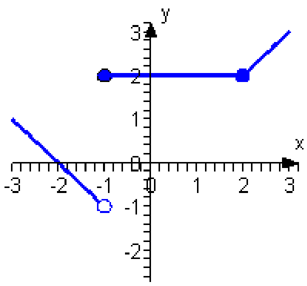

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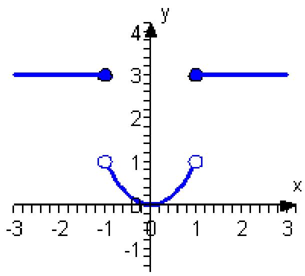

9. a) $D=[-2,2]$, $R=[-3,3]$, f is $\nearrow$ on $(-1,1)$, f is $\searrow$ on $(-2,-1)\cup(1,2)$

   b) $D=[-2,\infty)$, $R=[1,\infty)$, f is $\nearrow$ on $[-2,\infty)$

   c) $D=\mathbb{R}/\{0\}$, $R=(-\infty,-2]\cup[2,\infty)$, f is $\nearrow$ on $(-\infty,-1)\cup(1,\infty)$,
   f is $\searrow$ on $(-1,0)\cup(0,1)$

   d) $D=\mathbb{R}/\{0\}$, $R=\mathbb{R}$, f is $\searrow$ on $(-\infty,0)\cup(0,\infty)$

   e) $D=\mathbb{R}$, $R=(0,3]$, f is $\nearrow$ on $(-\infty,0)$, f is $\searrow$ on $(0,\infty)$

   f) $D=\mathbb{R}/\{\pm1\}$, $R=(-\infty,0]\cup(1,\infty)$, f is $\nearrow$ on $(-\infty,-1)\cup(-1,0)$,
   f is $\searrow$ on $(0,1)\cup(1,\infty)$