MATHEMATICS 201-009-50
Precalculus
Martin Huard
November 29, 2007

Test #4
SOLUTIONS

Answer all questions and show all your work. Exact answers are required whenever possible. Only the Sharp EL531 calculator is permitted.

Question 1 **(8 points)**

Without using a calculator, find the ***exact*** value of the following.

a) $\cos\left(-\frac{5\pi}{6}\right) = \frac{-\sqrt{3}}{2}$

b) $\cot\left(\frac{2\pi}{3}\right) = \frac{\cos\frac{2\pi}{3}}{\sin\frac{2\pi}{3}} = \frac{\frac{-1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

c) $\sec\left(\frac{13\pi}{4}\right) = \frac{1}{\cos\left(\frac{13\pi}{4}\right)} = \frac{1}{\frac{-\sqrt{2}}{2}} = -\sqrt{2}$

d) $\arcsin\left(\frac{-1}{2}\right) = -\frac{\pi}{6}$

Question 2 **(9 points)**

Without using a calculator, find the ***exact*** value of the following.

a) $\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{6}$

$= \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

or

$\sin\left(\frac{5\pi}{12}\right) = \sin\frac{\frac{5\pi}{6}}{2} = \sqrt{\frac{1-\cos\frac{5\pi}{6}}{2}}$

$= \sqrt{\frac{1-\frac{-\sqrt{3}}{2}}{2}}$

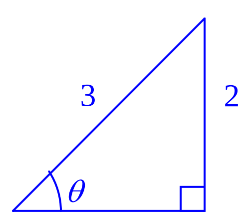
$= \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

b) $\sec\left(\arcsin\frac{2}{3}\right) = \sec\theta$

$= \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\arcsin\frac{2}{3} = \theta$

$\sin\theta = \frac{2}{3}$



$\sqrt{3^2-2^2} = \sqrt{5}$

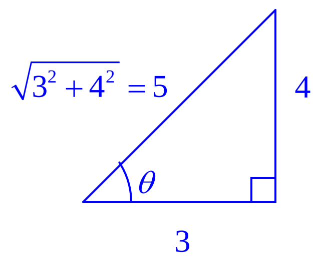
c) $\cos\frac{13\pi}{24}\cos\frac{7\pi}{24} = \frac{1}{2}\left(\cos\frac{\pi}{4}+\cos\frac{5\pi}{6}\right)$

$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}-\sqrt{3}}{4}$

Question 3 (9 points)

If $\tan\theta = \frac{4}{3}$ and θ is in quadrant III, find the ***exact*** value of



a) $\csc\theta = \frac{-5}{4}$

b) $\sin\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

$= \sqrt{\frac{1-\frac{3}{5}}{2}}$

$= \sqrt{\frac{2}{10}}$

$= \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$

If θ is in quadrant III, then $\frac{\theta}{2}$ is in quadrant II, hence $\sin\frac{\theta}{2} > 0$

c) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-3}{5}\right)^2 - \left(\frac{-4}{5}\right)^2$

$= \frac{9}{25} - \frac{16}{25} = \frac{-7}{25}$

Question 4 (6 points)

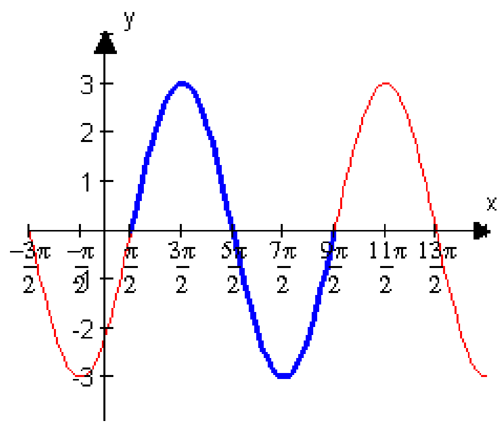
Find the amplitude, period, and phase shift of $y = 3\sin\left(\frac{x}{2} - \frac{\pi}{4}\right)$ and sketch the graph.

$y = 3\sin\left(\frac{x}{2} - \frac{\pi}{4}\right) = 3\sin\frac{1}{2}\left(x - \frac{\pi}{2}\right)$

Amplitude: 3

Period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$

Phase Shift: $\frac{\pi}{2}$

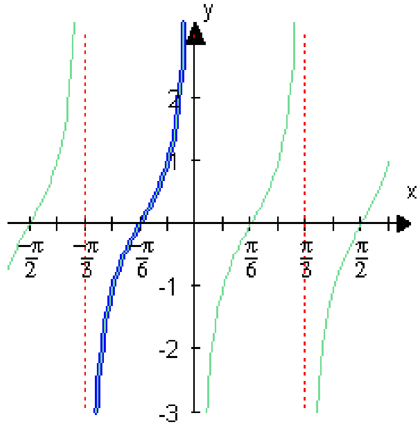


Question 5 (6 points)

Find the period and phase shift of $y = \tan 3\left(x + \frac{\pi}{6}\right)$ and sketch the graph.

Period: $\frac{\pi}{3}$

Phase Shift: $\frac{-\pi}{6}$



Question 6 (12 points)

Prove the identity.

a) $\dfrac{1+\tan^2\theta}{1-\tan^2\theta} = \dfrac{1}{\cos^2\theta - \sin^2\theta}$

$$\begin{aligned}
LS &= \frac{1+\tan^2\theta}{1-\tan^2\theta} \\
&= \frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}} \\
&= \frac{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}} \\
&= \frac{1}{\cancel{\cos^2\theta}} \frac{\cancel{\cos^2\theta}}{\cos^2\theta-\sin^2\theta} \\
&= \frac{1}{\cos^2\theta-\sin^2\theta} \\
&= RS
\end{aligned}$$

b) $\dfrac{\cos 3\theta - \cos 7\theta}{\sin 3\theta + \sin 7\theta} = \tan 2\theta$

$$\begin{aligned}
LS &= \frac{\cos 3\theta - \cos 7\theta}{\sin 3\theta + \sin 7\theta} \\
&= \frac{-2\cancel{\sin 5\theta}\sin(-2\theta)}{2\cancel{\sin 5\theta}\cos(-2\theta)} \quad \text{(Sum to Product formulas)} \\
&= \frac{2\sin 2\theta}{\cos 2\theta} \\
&= \tan 2\theta \\
&= RS
\end{aligned}$$