

MATHEMATICS 201-009-50

Precalculus

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Semester Review

1. Simplify each expression.

a) $\frac{6^2 x^3 y^{-3}}{12x^{-2}y}$

b) $\left(\frac{4a^3b^{-1}c}{a^{-2}b^4}\right)^{-2}$

c) $\left(\frac{2x^2y^{-1}}{x^{-3}}\right)^3 \left(\frac{x^{-9}y^6}{z^{12}}\right)^{-\frac{1}{3}}$

d) $\sqrt{\frac{18x^2}{y^8}}$

e) $\sqrt{18x^8} - x^2\sqrt{8x^4}$

f) $\sqrt[5]{t^{3n+1}}\sqrt[5]{t^{2n-1}}$

g) $(3x-2)^2 - (2x-5)$

h) $\frac{1}{x-1} + \frac{1-x}{x^2+x+1}$

i) $\frac{x^3-8}{x^2-5x+6}$

j) $\frac{3x}{x+2} - \frac{4x^2-5}{2x^2+3x-2}$

k) $\frac{\frac{1}{2x-3} - \frac{1}{2x+3}}{\frac{1}{2x} - \frac{1}{2x+3}}$

l) $\frac{x^2-4}{x^4-2x^2-8} \cdot \frac{x^2+2}{x^2}$

m) $\frac{4x-6}{(x-1)^2} \div \frac{2x^2-3x}{x^2+2x-3}$

n) $\frac{5}{x} + \frac{1}{x+2} + \frac{3}{x^2+x-2}$

o) $\frac{(x+3)^{\frac{5}{3}} - 12x(x+3)^{-\frac{1}{3}}}{x^2-9}$

2. Rationalize the denominator.

a) $\frac{1}{2-\sqrt{3}}$

b) $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

3. Factor completely.

a) $x^2 - 12x + 36$

b) $8x^3 + 27$

c) $12x^3 + 7x^2 + x$

d) $x(x+2)^{\frac{1}{2}} + (x+2)^{\frac{1}{2}}$

e) $x^3 + 3x^2 - 4x - 12$

f) $6(2x-1)^2(x+5)^5 + 2(2x-1)^3(x+5)^4$

g) $8x^2(x-1)^{-3}(x+2)^{\frac{3}{4}} - 4x(x-1)^{-4}(x+2)^{\frac{5}{4}}$

4. Find all real solutions of each equation.

a) $\frac{4x-3}{6} + \frac{x}{4} = x-2$

b) $\frac{5}{x-2} = \frac{13}{2x-3}$

c) $x^2 - 13x + 30 = 0$

d) $x^2 + 6x - 3 = 0$

e) $\frac{x}{x+2} - \frac{5}{x} = \frac{2x-2}{x^2+2x}$

f) $(x+4)^{\frac{1}{2}} + 5x(x+4)^{\frac{3}{2}} = 0$

g) $\sqrt{x-2} - 8 = 0$

h) $x^4 - 5x^2 + 6 = 0$

i) $5x^4 - 12x^3 = 0$

j) $|2x-5|=9$

5. A rectangular garden is to be twice as long as it is wide. What should be its dimension if it is to have a total area of 80 m²?

6. Solve each inequality. Give your answer in interval notation.

a) $9x - 8 \leq 7x + 16$ b) $|2x - 3| < 5$ c) $x^2 - 2x - 3 \geq 0$ d) $\frac{2}{x+1} \leq \frac{3}{x-1}$

7. A car radiator contains 10 liters of a 30% antifreeze solution. How many liters will have to be replaced with pure antifreeze if the resulting solution is to be 50% antifreeze.

8. Let $A(-2,5)$ and $B(1,3)$ be points in the plane.

- Find the length and midpoint of the segment AB .
- Find an equation for the line passing through the points A and B .
- Find an equation for the line perpendicular to the segment AB and passing through the midpoint of the segment AB .
- Find the equation for the circle having A and B as the endpoints of a diameter.

9. Find the radius and center for the circle $x^2 + 6x + y^2 - 10y + 18 = 0$ and sketch the graph.

10. Find the domain of the following functions and determine whether they are even or odd.

a) $f(x) = \frac{x}{x^2 - 4}$ b) $f(x) = \frac{\sqrt{x-2} + 5}{x-10}$
 c) $f(x) = \sqrt{x^2 - 9}$ d) $f(x) = \sqrt{6 + x - x^2}$

11. Let $f(x) = x^2 + x - 1$. Find

a) $f(2t+1)$ b) $f\left(\frac{x}{x+1}\right)$ c) $\frac{f(x+h) - f(x)}{h}$

12. Let $f(x) = \frac{x+2}{x+4}$. Find

a) $f(x^2 - 1)$ b) $f\left(\frac{2}{x+3}\right)$ c) $\frac{f(x+h) - f(x)}{h}$

13. Find the x and y intercepts of the given function and sketch the graph.

a) $f(x) = \sqrt{x+1} - 2$ b) $f(x) = x^3 + 8$ c) $f(x) = 3|x-2| + 5$
 d) $f(x) = -\sqrt{4-x} + 2$ e) $f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 2 \\ 3x - 1 & \text{if } x > 2 \end{cases}$ f) $f(x) = \begin{cases} 9 - x^2 & \text{if } x < -2 \\ \sqrt{x+3} & \text{if } x \geq -2 \end{cases}$

14. For the given function,

- find the vertex;
- find the intercepts;
- find maximum or minimum value that f takes;
- find the domain and range;
- sketch the graph.

a) $f(x) = x^2 - 4x + 3$ b) $f(x) = -2x^2 + 12x + 14$

15. A sports center has 25 skidoos to rent. The owner finds that if he charges \$60 per day, all of his skidoos will be rented. However, for each \$4 increase in the price, he will rent one skidoo less. What price should he charge to maximize his revenue?

16. Let $f(x) = x^2 + x - 2$ and $g(x) = \sqrt{x} + 1$. Find the following function, and state the domain of each.

a) $f + g$

b) $f \cdot g$

c) $f \circ g$

d) $g \circ f$

e) $f \circ f$

f) $g \circ g$

17. For the given function,

i) find the inverse f^{-1} of f ;

ii) find the domain and range of f and f^{-1} ;

iii) verify that $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$;

iv) sketch the graph of f and f^{-1} .

a) $f(x) = \sqrt{x-1} + 3$

b) $f(x) = (x+2)^2 - 1 \quad x \geq -2$

18. Assuming that f is invertible, find $f^{-1}(2)$ if $f(3) = 2$.

19. Divide the following.

a) $\frac{x^4 - 3x^2 + x - 5}{x^2 + 2x}$

b) $\frac{x^6 - 3x^5 + x^2 - 2}{x^3 + 3x + 1}$

c) $\frac{x^8 - 1}{x^7 + 1}$

20. Factor the following polynomials.

a) $p(x) = x^3 - 4x^2 + x + 6$

b) $p(x) = x^3 - 4x^2 - 11x + 30$

c) $p(x) = x^4 - 2x^3 - 11x^2 + 12x + 36$

d) $p(x) = 2x^3 + 3x^2 - 23x - 12$

21. Solve the following equations. (*Hint*: find all rational zeros.)

a) $x^3 - 5x^2 - 4x + 20 = 0$

b) $x^3 + 2x^2 - 2x - 3 = 0$

c) $2x^3 + 3x^2 - 8x + 3 = 0$

d) $4x^3 - 7x + 3 = 0$

22. Find the domain, range, intercepts and asymptotes (if any) for the following functions, and sketch the graph.

a) $f(x) = 3^x - 9$

b) $f(x) = -e^{x+1}$

c) $f(x) = -\left(\frac{1}{2}\right)^{x+2} + 4$

d) $f(x) = \log_3(x+1) - 2$

e) $f(x) = -\ln(x) + 1$

f) $f(x) = \log_2(4-x) + 1$

23. Write as a single logarithm.

a) $2\log_3 x - 5\log_3 y^2$

b) $5\log_2 x^{-2} - 3\log_2(x+1) + 1$

24. Write the expression as a sum, difference, and/or multiple of logarithms.

a) $\log_3 \frac{3x^5}{y^2}$

b) $\ln \frac{x-1}{x^3}$

25. Find the solution of the equation.

a) $3e^{-5x} = 132$

b) $3^{2x+1} - 4 = 7$

c) $e^{2x} - 7e^x + 10 = 0$

d) $2 \ln 4x = 15$

e) $4 \log_3(x+5) - 2 = 7$

f) $\log x + \log(x+1) = \log 12$

26. A culture contains 10 000 bacteria initially. After an hour the bacteria count is 25 000.

a) Find the population after 3 hours.

b) How long will it take for the population to double?

27. A man invests \$5000 in a mutual fund which pays 8% per year, compounded monthly.

a) How much money will the man have in 5 years?

b) How long would it take for the amount to triple?

28. Find the central angle θ in a circle of radius 4m that subtends an arc of length 5π m. Give the answer in degrees and in radians.

29. Find the area of a sector with central angle 150° in a circle of radius 4m.

30. Find the **exact value** of the following expressions.

a) $\cos \frac{2\pi}{3}$

b) $\sec \frac{3\pi}{4}$

c) $\cot \frac{13\pi}{3}$

d) $\tan \frac{\pi}{12}$

e) $\sin \frac{7\pi}{12}$

f) $2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$

g) $\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8}$

h) $\sin(\arccos \frac{1}{5})$

i) $\sec(\arctan 4)$

j) $\csc(\arcsin \frac{3}{7})$

k) $\arccos(\cos \frac{7\pi}{4})$

l) $\arctan(\tan \frac{5\pi}{4})$

m) $\sin(\arcsin \frac{2}{3} + \arccos \frac{1}{3})$

n) $\cos(2 \arctan \frac{4}{3})$

31. If $\cot \theta = -\frac{3}{4}$ and θ is in quadrant II, find

a) $\sin \theta$

b) $\cos \theta$

c) $\tan \theta$

d) $\sec \theta$

e) $\sin 2\theta$

f) $\cos 2\theta$

g) $\sin \frac{\theta}{2}$

h) $\cos \frac{\theta}{2}$

i) $\tan 2\theta$

j) $\csc \frac{\theta}{2}$

k) $\sec 2\theta$

l) $\cot \frac{\theta}{2}$

32. If $\sec \theta = \frac{6}{5}$ and $\tan \theta < 0$, find

a) $\sin \theta$

b) $\cos \theta$

c) $\cot \theta$

d) $\csc \theta$

e) $\sin 2\theta$

f) $\cos 2\theta$

g) $\sin \frac{\theta}{2}$

h) $\cos \frac{\theta}{2}$

i) $\tan 2\theta$

j) $\csc \frac{\theta}{2}$

k) $\sec 2\theta$

l) $\cot \frac{\theta}{2}$

33. If $\sin \alpha = \frac{-5}{13}$ and $\tan \beta = \frac{-3}{4}$, where α is in quadrant III and β is in quadrant II, find the exact value of the trigonometric function.

a) $\sin(\alpha + \beta)$

b) $\sin(\alpha - \beta)$

c) $\cos(\alpha + \beta)$

d) $\cos(\alpha - \beta)$

e) $\tan(\alpha + \beta)$

f) $\csc(\alpha + \beta)$

g) $\sec(\alpha - \beta)$

h) $\cot(\alpha - \beta)$

34. Write the first expression in terms of the second if the terminal point determined by θ is in the given quadrant.

- a) $\cos \theta$ in terms of $\tan \theta$ if θ in quadrant III.
 b) $\sin \theta$ in terms of $\sec \theta$ if θ is in quadrant II.

35. Find the amplitude, period and phase shift and sketch the graph.

a) $f(x) = 2 \cos 3(x - \frac{\pi}{3})$ b) $f(x) = -4 \sin(\frac{x}{3} + \pi)$ c) $f(x) = 2 \sin(\pi x - \frac{\pi}{4})$

36. Find the period and phase shift and sketch the graph.

a) $f(x) = 3 \csc(x - \frac{\pi}{2})$ b) $f(x) = 2 \sec(\pi x - \frac{\pi}{2})$ c) $f(x) = 5 \sec \frac{1}{2}(x + \frac{\pi}{2})$
 d) $f(x) = \tan 2(x + \frac{\pi}{3})$ e) $f(x) = \tan(\pi x - \frac{\pi}{3})$ f) $f(x) = -\cot(\frac{x}{2} + \frac{\pi}{4})$

37. Verify each identity.

a) $\frac{\csc \theta + \sec \theta}{\sin \theta + \cos \theta} = \cot \theta + \tan \theta$

b) $\frac{\tan^2 \theta + 1}{\sec \theta \tan \theta} = \csc \theta$

c) $\frac{\sin \theta + 1}{\sin \theta} = \frac{\cot^2 \theta}{\csc \theta - 1}$

d) $\frac{\cos 4\theta + \cos 2\theta}{\sin 4\theta + \sin 2\theta} = \cot 3\theta$

e) $\csc 2\theta = \frac{\csc \theta}{2 \cos \theta}$

f) $\cos^4 \theta - \sin^4 \theta = \cos 2\theta$

g) $\sec^2 \frac{\theta}{2} = \frac{2(1 - \cos \theta)}{\sin^2 \theta}$

h) $\tan \theta = \frac{1 - \cos 2\theta}{\sin 2\theta}$

i) $\sec(A - B) = \frac{\csc A \csc B}{1 + \cot A \cot B}$

j) $\cot\left(\frac{A + B}{2}\right) = \frac{\sin A - \sin B}{\cos B - \cos A}$

38. a) Write $\cos 2x \sin 5x$ as a sum of trigonometric functions.

b) Write $\cos 2x + \cos 5x$ as a product of trigonometric functions.

39. Rewrite the expression as an algebraic function of x .

a) $\sin(\arctan x)$

b) $\cot(\arccos x)$

40. Solve each equation on the interval $[0, 2\pi)$.

a) $\tan \theta = -\frac{\sqrt{3}}{3}$

b) $\sin 3\theta = -1$

c) $\sec \frac{3\theta}{2} = -2$

d) $2 \cos^2 \theta - \cos \theta - 1 = 0$

e) $\tan \theta = 2 \sin \theta$

f) $\sin 2\theta + \sin 4\theta = 0$

g) $\tan^2 \theta = \frac{3}{2} \sec \theta$

41. Solve the following triangles.

a) $A = 20^\circ$, $a = 5$, $b = 8$.

b) $A = 24.3^\circ$, $C = 54.6^\circ$, $c = 2.6$

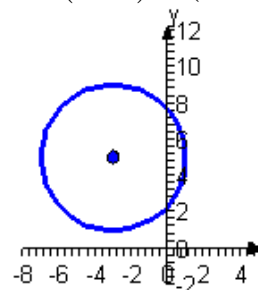
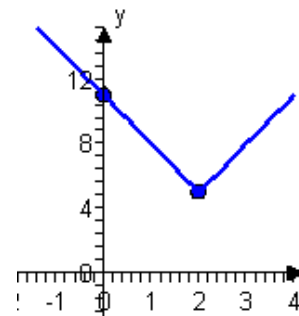
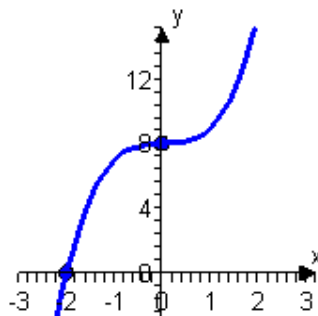
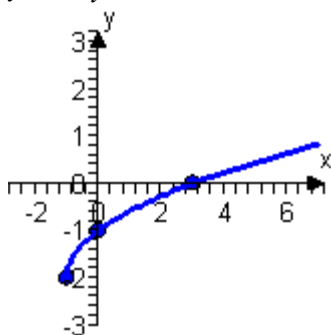
c) $a = 55$, $b = 52$, $c = 72$

d) $A = 55^\circ$, $b = 3$, $c = 10$

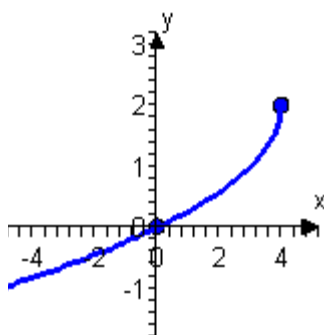
42. From a point A on the ground, the angle of elevation to the top of a skyscraper is 39.2° . From a point B , 500m from point A , the angle of elevation is 62.5° . Find the height of the skyscraper.

ANSWERS

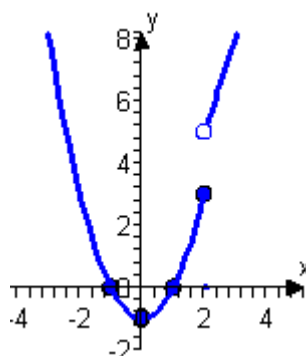
1. a) $\frac{3x^5}{y^4}$ b) $\frac{b^{10}}{16a^{10}c^2}$ c) $\frac{8x^{18}c^4}{y^5}$ d) $\frac{3\sqrt{2}|x|}{y^4}$ e) $\sqrt{2}x^4$ f) t^n
- g) $9x^2 - 14x + 9$ h) $\frac{3x}{(x-1)(x^2+x+1)}$ i) $\frac{x^2+2x+4}{x-3}$ j) $\frac{2x^2-3x+5}{(x+2)(2x-1)}$ k) $\frac{4x}{2x-3}$
- l) $\frac{1}{x^2}$ m) $\frac{2(x+3)}{x(x-1)}$ n) $\frac{6x-5}{x(x-1)}$ o) $\frac{x-3}{(x+3)^{\frac{4}{3}}}$
2. a) $2 + \sqrt{3}$ b) $6 + \sqrt{35}$
3. a) $(x-6)^2$ b) $(2x+3)(4x^2-6x+9)$ c) $x(3x+1)(4x+1)$ d) $\frac{2(x+1)}{\sqrt{x+2}}$
- e) $(x+3)(x-2)(x+2)$ f) $2(5x+14)(2x-1)^2(x+5)^4$ g) $\frac{4x(x^2-6x-4)}{(x-3)^4(x+2)^{\frac{3}{4}}}$
4. a) 18 b) $\frac{11}{3}$ c) 3, 10 d) $-3 \pm 2\sqrt{3}$ e) -1, 8 f) $-4, -2 \pm \frac{\sqrt{95}}{5}$
- g) 66 h) $\pm\sqrt{2}, \pm\sqrt{3}$ i) $0, \frac{12}{5}$ j) -2, 7
5. $2\sqrt{10}$ meters
6. a) $(-\infty, 12)$ b) $(-1, 4)$ c) $(-\infty, -1] \cup [3, \infty)$ d) $[-5, -1) \cup (1, \infty)$
7. $\frac{20}{7}$ liters
8. a) $\sqrt{13}$ M $(\frac{-1}{2}, 4)$ b) $y = -\frac{2}{3}x + \frac{11}{3}$ c) $y = \frac{3}{2}x + \frac{19}{4}$ d) $(x + \frac{1}{2})^2 + (y - 4)^2 = \frac{13}{4}$
9. Radius: 4 Center: $(-3, 5)$
10. a) $\mathbb{R} / \{-2, 2\}$, odd b) $[2, 10) \cup (10, \infty)$ neither
- c) $(-\infty, -3] \cup [3, \infty)$, even d) $[-2, 3]$, neither
11. a) $4t^2 + 6t + 1$ b) $\frac{x^2-x-1}{(x+1)^2}$ c) $2x + h + 1$
12. a) $\frac{x^2+1}{x^2+3}$ b) $\frac{x+4}{2x+7}$ c) $\frac{2}{(x+4)(x+4+h)}$
13. a) x-int: $x = 3$
y-int: $y = -1$
- b) x-int: $x = -2$
y-int: $y = 8$
- c) x-int: none
y-int: $y = 11$



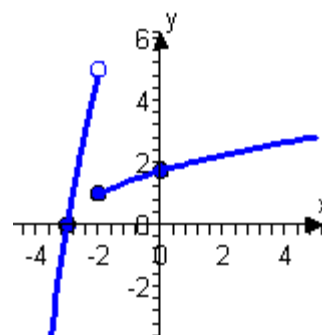
- d) x -int: $x=0$
 y -int: $y=0$



- e) x -int: $x=\pm 1$
 y -int: $y=-1$



- f) x -int: $x=-3$
 y -int: $y=\sqrt{3}$



14. a) i) $(2, -1)$

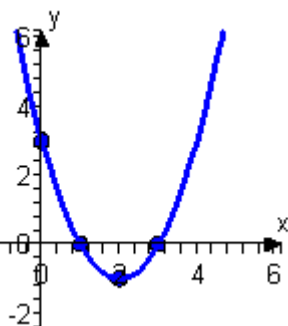
ii) x -int: 1, 3

y -int: 3

iii) Min : -1

iv) D: \mathbb{R}

R : $[-1, \infty)$



- b) i) $(3, 32)$

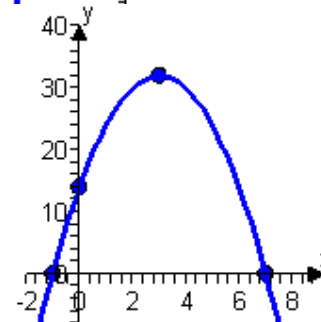
ii) x -int: -1, 7

y -int: 14

iii) Max : 32

iv) D: \mathbb{R}

R : $(-\infty, 32]$



15. \$80 per day

16. a) $x^2 + x + \sqrt{x} - 1$

b) $x^{\frac{5}{2}} + x^2 + x^{\frac{3}{2}} + x - 2x^{\frac{1}{2}} - 2$

c) $x + 3\sqrt{x}$

d) $\sqrt{x^2 + x - 2} + 1$

e) $x^4 + 2x^3 - 2x^2 - 3x$

f) $\sqrt{\sqrt{x+1}} + 1$

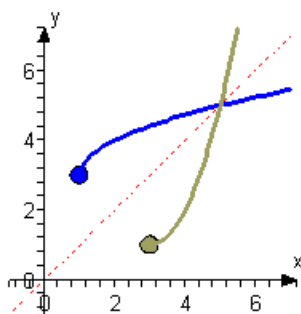
17. a) i) $f^{-1}(x) = (x-3)^2 + 1 \quad x \geq 3$

ii) D. of f : $[1, \infty)$

R. of f : $[3, \infty)$

D. of f^{-1} : $[3, \infty)$

R. of f^{-1} : $[1, \infty)$



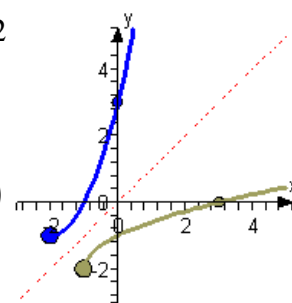
- b) i) $f^{-1}(x) = \sqrt{x+1} - 2$

ii) D. of f : $[-1, \infty)$

R. of f : $[-2, \infty)$

D. of f^{-1} : $[-2, \infty)$

R. of f^{-1} : $[-1, \infty)$



18. $f^{-1}(2) = 3$

19. a) $x^2 - 2x + 1 - \frac{x+5}{x^2+2x}$

b) $x^3 - 3x^2 - 3x + 8 + \frac{13x^2 - 21x - 10}{x^3 + 3x + 1}$

f) $x - \frac{x+1}{x^7+1}$

20. a) $(x-3)(x-2)(x+1)$

b) $(x-5)(x-2)(x+3)$

c) $(x+2)^2(x-3)^2$

d) $(2x+1)(x-3)(x+4)$

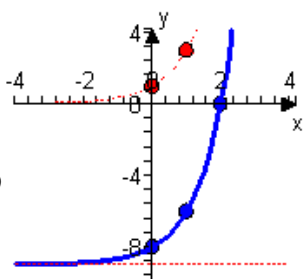
21. a) -2, 2, 5

b) -1, $\frac{-1 \pm \frac{1}{2}\sqrt{13}}$

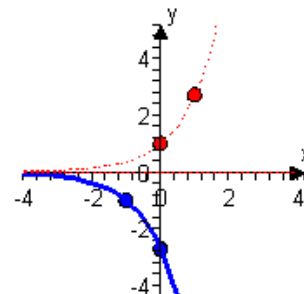
c) -3, 1, $\frac{1}{2}$

d) $\frac{-3}{2}$, 1, $\frac{1}{2}$

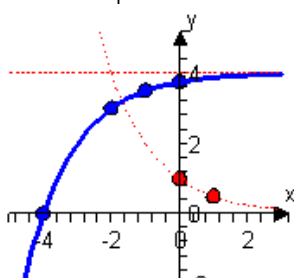
22. a) $D: \mathbb{R}$
 $R: (-9, \infty)$
 $x\text{-int}: 2$
 $y\text{-int}: -8$
 $H.A.: y = -9$



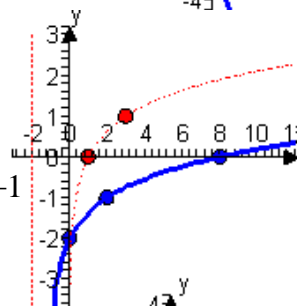
- b) $D: \mathbb{R}$
 $R: (-\infty, 0)$
 $x\text{-int}: \text{none}$
 $y\text{-int}: -e$
 $H.A.: y = 0$



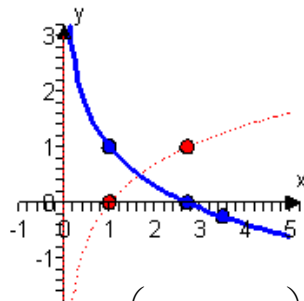
- c) $D: \mathbb{R}$
 $R: (-\infty, 4)$
 $x\text{-int}: -4$
 $y\text{-int}: \frac{15}{4}$
 $H.A.: y = 4$



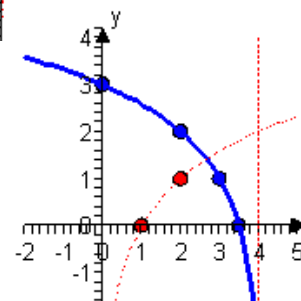
- $D: (-1, \infty)$
 $R: \mathbb{R}$
 $x\text{-int}: -2$
 $y\text{-int}: 8$
 $V.A.: x = -1$



- e) $D: (0, \infty)$
 $R: \mathbb{R}$
 $x\text{-int}: e$
 $y\text{-int}: \text{none}$
 $V.A.: x = 0$



- f) $D: (-\infty, 4)$
 $R: \mathbb{R}$
 $x\text{-int}: \frac{1}{2}$
 $y\text{-int}: 3$
 $V.A.: x = 4$



23. a) $\log_3\left(\frac{x^2}{y^{10}}\right)$ b) $\log_2\left(\frac{2}{x^{10}(x+1)^3}\right)$

24. a) $1 + 5\log_3 x - 2\log_3 y$

b) $\ln(x-1) - 3\ln x$

25. a) $\frac{-\ln 44}{5}$ b) $\frac{1}{2}\log_3 11 - \frac{1}{2}$ c) $\ln 2, \ln 5$ d) $\frac{1}{4}e^{\frac{15}{2}}$ e) $3^{\frac{9}{4}} - 5$ f) 3

26. a) 19531250 b) 0.274 hours or 16.5 minutes

27. a) \$7449.23 b) 13.8 years

28. $\frac{5\pi}{4}$ rad or 135°

29. $\frac{20\pi}{3} \text{ m}^2$

30. a) $-\frac{1}{2}$ b) $-\sqrt{2}$ c) $\frac{\sqrt{3}}{3}$ d) $2 - \sqrt{3}$ e) $\frac{\sqrt{6} + \sqrt{2}}{4}$ f) $\frac{1}{2}$

g) $\frac{\sqrt{2}}{2}$ h) $\frac{2\sqrt{6}}{5}$ i) $\sqrt{17}$ j) $\frac{7}{3}$ k) $\frac{\pi}{4}$ l) $\frac{\pi}{4}$

m) $\frac{2}{9} + \frac{2\sqrt{10}}{9}$ n) $\frac{-7}{25}$

31. a) $\frac{4}{5}$ b) $\frac{-3}{5}$ c) $\frac{-4}{3}$ d) $\frac{-5}{3}$ e) $\frac{-24}{25}$ f) $\frac{-7}{25}$

g) $\frac{2\sqrt{5}}{5}$ h) $\frac{\sqrt{5}}{5}$ i) $\frac{24}{7}$ j) $\frac{\sqrt{5}}{2}$ k) $\frac{-25}{7}$ l) $\frac{1}{2}$

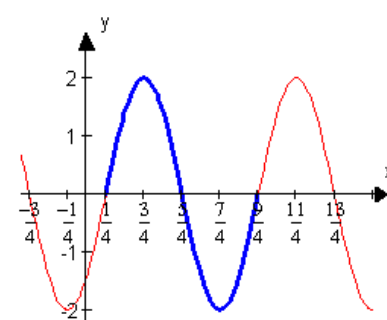
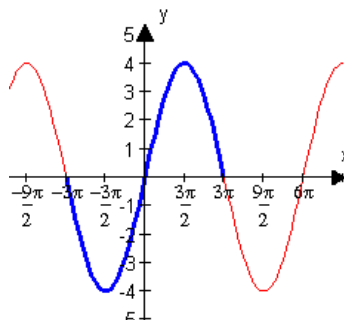
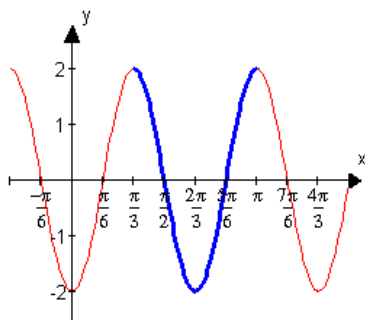
32. a) $\frac{-\sqrt{11}}{6}$ b) $\frac{5}{6}$ c) $\frac{-5\sqrt{11}}{11}$ d) $\frac{-6\sqrt{11}}{11}$ e) $\frac{-5\sqrt{11}}{18}$ f) $\frac{7}{18}$

g) $\frac{\sqrt{3}}{6}$ h) $\frac{-\sqrt{33}}{6}$ i) $\frac{-5\sqrt{11}}{7}$ j) $2\sqrt{3}$ k) $\frac{18}{7}$ l) $-\sqrt{11}$

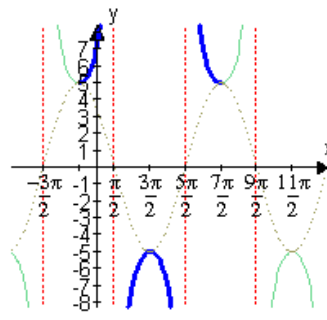
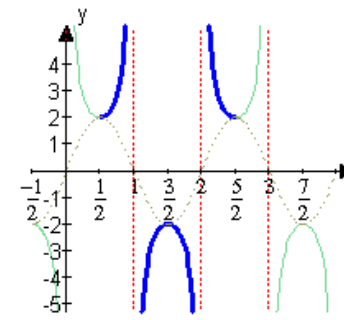
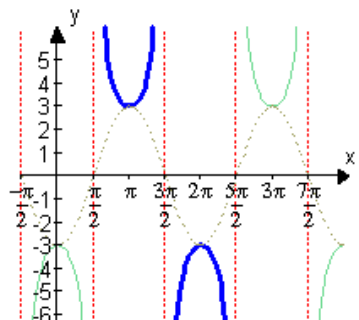
33. a) $\frac{-16}{65}$ b) $\frac{56}{65}$ c) $\frac{63}{65}$ d) $\frac{33}{65}$ e) $\frac{-16}{63}$ f) $\frac{-65}{16}$
 g) $\frac{65}{33}$ h) $\frac{33}{56}$

34. a) $\cos \theta = \frac{-1}{\sqrt{1+\tan^2 \theta}}$ b) $\sin \theta = \frac{-\sqrt{\sec^2 \theta - 1}}{\sec \theta}$

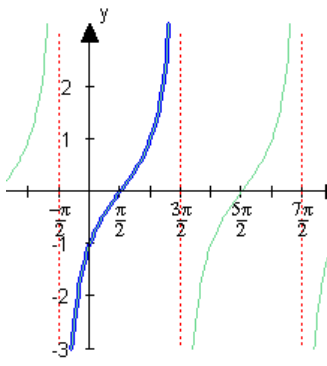
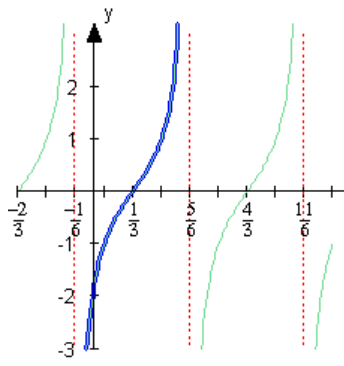
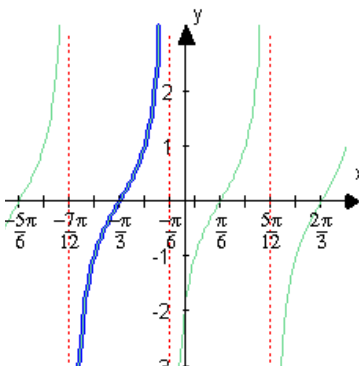
35. a) Amplitude: 2
 Period: $\frac{2\pi}{3}$
 Phase Shift: $\frac{\pi}{3}$ b) Amplitude: 4
 Period: 6π
 Phase Shift: -3π c) Amplitude: 2
 Period: 2
 Phase Shift: $\frac{1}{4}$



36. a) Period: 2π
 Phase Shift: $\frac{\pi}{2}$ b) Period: 2
 Phase Shift: $\frac{1}{2}$ c) Period: 4π
 Phase Shift: $-\frac{\pi}{2}$



- d) Period: $\frac{\pi}{2}$
 Phase Shift: $-\frac{\pi}{3}$ e) Period: 1
 Phase Shift: $\frac{1}{3}$ f) Period: 2π
 Phase Shift: $-\frac{1}{2}$



37. a) $LS = \frac{\csc \theta + \sec \theta}{\sin \theta + \cos \theta} = \frac{\frac{1}{\sin \theta} + \frac{1}{\cos \theta}}{\sin \theta + \cos \theta} = \frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta} \cdot \frac{1}{\sin \theta + \cos \theta} = \frac{1}{\sin \theta \cos \theta}$
 $RS = \cot \theta + \tan \theta = \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} = LS$
 b) $LS = \frac{\tan^2 \theta + 1}{\sec \theta \tan \theta} = \frac{\sec^2 \theta}{\sec \theta \tan \theta} = \frac{\sec \theta}{\tan \theta} = \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta}} = \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta} = \csc \theta = RS$

$$c) RS = \frac{\cot^2 \theta}{\csc \theta - 1} = \frac{\csc^2 \theta - 1}{\csc \theta - 1} = \frac{(\csc \theta + 1)(\csc \theta - 1)}{\csc \theta - 1} = \csc \theta + 1 = \frac{1}{\sin \theta} + 1 = \frac{\sin \theta + 1}{\sin \theta} = LS$$

$$d) LS = \frac{\cos 4\theta + \cos 2\theta}{\sin 4\theta + \sin 2\theta} = \frac{2\cos 3\theta \cos \theta}{2\sin 3\theta \cos \theta} = \cot 3\theta = RS$$

$$e) LS = \csc 2\theta = \frac{1}{\sin 2\theta} = \frac{1}{2\sin \theta \cos \theta} = \frac{\csc \theta}{2\cos \theta} = RS$$

$$f) LS = \cos^4 \theta - \sin^4 \theta = (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) = \cos 2\theta = RS$$

$$g) LS = \sec^2 \frac{\theta}{2} = \frac{1}{\cos^2 \frac{\theta}{2}} = \frac{1}{\frac{1+\cos \theta}{2}} = \frac{2}{1+\cos \theta}$$

$$RS = \frac{2(1-\cos \theta)}{\sin^2 \theta} = \frac{2(1-\cos \theta)}{1-\cos^2 \theta} = \frac{2(1-\cos \theta)}{(1-\cos \theta)(1+\cos \theta)} = \frac{2}{1+\cos \theta} = LS$$

$$h) RS = \frac{1-\cos 2\theta}{\sin 2\theta} = \frac{1-(1-2\sin^2 \theta)}{2\sin \theta \cos \theta} = \frac{2\sin^2 \theta}{2\sin \theta \cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta = LS$$

$$i) LS = \sec(A-B) = \frac{1}{\cos(A-B)} = \frac{1}{\cos A \cos B + \sin A \sin B} \cdot \frac{\frac{1}{\sin A \sin B}}{\frac{1}{\sin A \sin B}} = \frac{\frac{1}{\sin A \sin B}}{\frac{\cos A \cos B + \sin A \sin B}{\sin A \sin B}} = \frac{\csc A \csc B}{\cot A \cot B + 1} = RS$$

$$j) RS = \frac{\sin A - \sin B}{\cos B - \cos A} = \frac{2\cos \frac{A+B}{2} \sin \frac{A-B}{2}}{2\sin \frac{A+B}{2} \sin \frac{A-B}{2}} = \cot \frac{A+B}{2} = LS$$

$$38. a) \frac{1}{2} \sin 7x + \frac{1}{2} \sin 3x$$

$$b) 2 \cos \frac{7x}{2} \cos \frac{3x}{2}$$

$$39. a) \frac{x}{\sqrt{x^2 + 1}}$$

$$b) \frac{x}{\sqrt{1-x^2}}$$

$$40. a) \frac{5\pi}{6}, \frac{11\pi}{6}$$

$$b) \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$c) \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}$$

$$d) 0, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$e) 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$$f) 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$$

$$g) \frac{\pi}{3}, \frac{5\pi}{3}$$

$$41. a) B_1 = 33.2^\circ, C_1 = 126.8^\circ, c_1 = 11.7$$

$$B_2 = 146.8^\circ, C_2 = 13.2^\circ, c_2 = 3.23$$

$$b) B = 101.1^\circ, a = 1.31, b = 3.3$$

$$c) A = 49.5^\circ, B = 46.0^\circ, C = 84.5^\circ$$

$$d) B = 16.5^\circ, C = 108.5^\circ, a = 8.64$$

$$42. 708.7 \text{ m}$$