

Quiz #9 SOLUTIONS

Question 1 (1 point)

Let $f(x) = x^3 + x^2 - x + 2$. Find x if $f^{-1}(x) = 1$.

$$x = f(1) = 1^3 + 1^2 - 1 + 2 = 3$$

Question 2 (9 points)

Consider the function $f(x) = \sqrt{x+4} - 3$.

a) Find the inverse of $f(x)$.

$$y = \sqrt{x+4} - 3$$

$$\therefore f^{-1}(x) = (x+3)^2 - 4$$

$$x = \sqrt{y+4} - 3$$

$$(x+3)^2 = y+4$$

$$y = (x+3)^2 - 4$$

b) Verify that $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$.

$$(f^{-1} \circ f)(x) = f^{-1}(f(x))$$

$$(f \circ f^{-1})(x) = f(f^{-1}(x))$$

$$= f^{-1}(\sqrt{x+4} - 3)$$

$$= f((x+3)^2 - 4)$$

$$= (\sqrt{x+4} - \cancel{3} + \cancel{3})^2 - 4$$

$$= \sqrt{(x+3)^2 - \cancel{4} + \cancel{4}} - 3$$

$$= x+4-4$$

$$= |x+3| - 3$$

$$= x \quad x \geq -4$$

$$= x+3-3 \quad x \geq -3$$

$$= x$$

c) Find the domain and range of $f(x)$ and $f^{-1}(x)$.

$$\text{Domain of } f: [-4, \infty) \quad \text{Range of } f: [-3, \infty)$$

$$\text{Domain of } f^{-1}: [-3, \infty) \quad \text{Range of } f^{-1}: [-4, \infty)$$

d) Sketch the graph of $f(x)$ and $f^{-1}(x)$.

