

## MATHEMATICS 201-009-50

Precalculus  
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# IX – Quadratic Functions

1. Sketch the graph of the given parabola by finding the vertex as well as the intercepts.

- |                           |                                    |                            |
|---------------------------|------------------------------------|----------------------------|
| a) $f(x) = x^2 - 5$       | b) $f(x) = (x+2)^2$                | c) $f(x) = x^2 + 6x$       |
| d) $f(x) = x^2 - 6x + 8$  | e) $f(x) = -x^2 - 2x - 1$          | f) $f(x) = 2x^2 + 8x - 5$  |
| g) $f(x) = x^2 + x + 1$   | h) $f(x) = -3x^2 + 3x + 1$         | i) $f(x) = 3x^2 - 12x + 9$ |
| j) $f(x) = 2x^2 + 5x + 4$ | k) $f(x) = \frac{1}{2}x^2 + x - 3$ | l) $f(x) = -2x^2 + 3x - 5$ |

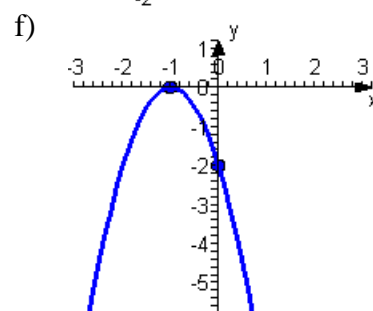
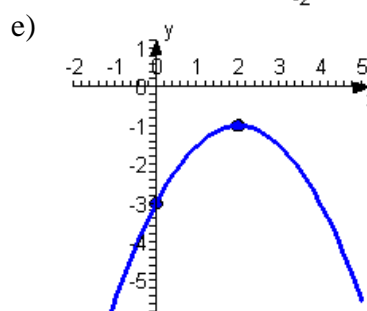
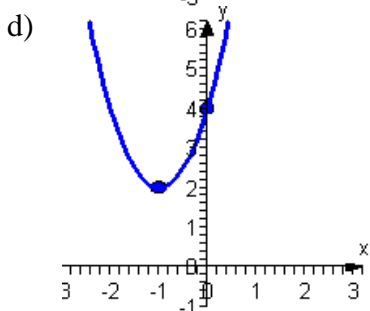
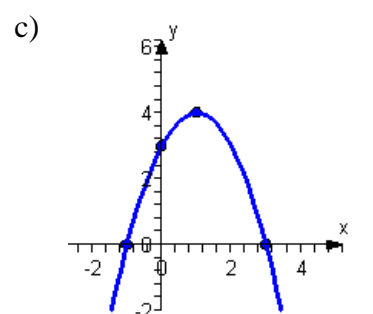
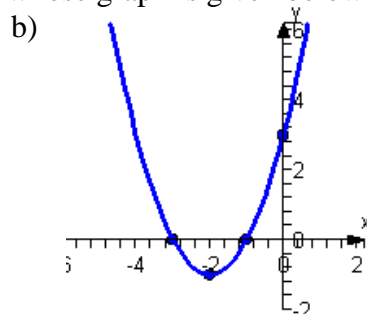
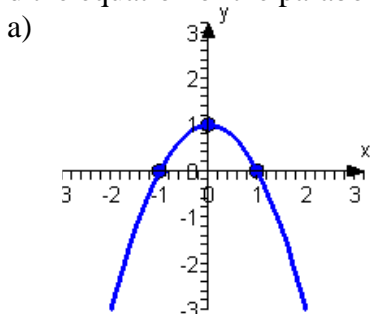
2. Find the maximum or minimum value of the function.

- |                                      |                          |                          |
|--------------------------------------|--------------------------|--------------------------|
| a) $f(x) = x^2 + 10x + 10$           | b) $f(x) = 20x^2 - 600x$ | c) $f(x) = 1 + 5x - x^2$ |
| d) $f(x) = \frac{-1}{3}x^2 + 2x - 5$ | e) $f(t) = 30t - t^2$    | f) $R(p) = p(50 - p)$    |

3. Find the equation of the parabola with the indicated vertex and whose graph passes through the given point.

- Vertex  $(1, -1)$  passing through  $(0, 0)$ .
- Vertex  $(2, -1)$  passing through  $(0, 5)$ .
- Vertex  $(2, 3)$  passing through  $(4, 0)$ .
- Vertex  $(-4, -1)$  passing through  $(-1, -2)$ .

4. Find the equation of the parabola whose graph is given below



5. Find two numbers whose difference is 200 and whose product is as small as possible.
6. A farmer wishes to enclose a rectangular plot by a fence and then divide it into two plots by another fence parallel to one of the sides. What are the dimensions of the largest area that can be enclosed by using a total of 1800 meters of fencing?
7. If a ball is thrown into the air with a velocity of 10 m/s, its height (in meters) after  $t$  seconds is given by

$$y = 10t - 4.9t^2$$

- a) Find the maximum height the ball will reach.
- b) How long will it take to reach this maximum height?
8. A projectile is fired from a cliff 200 meters above the water at an inclination of  $45^\circ$  to the horizontal, with a muzzle velocity of 28 meters per second. The height  $h$  of the projectile above the water is given by

$$h(x) = \frac{-9.8x^2}{(28)^2} + x + 200$$

where  $x$  is the horizontal distance (in m) of the projectile from the cliff.

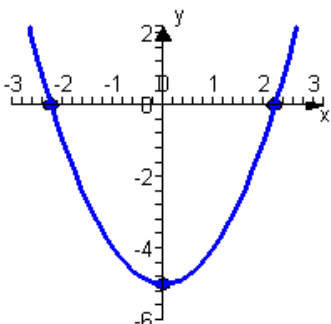
- a) Find the maximum height of the projectile.
- b) How far from the base of the cliff will the projectile hit the water?
9. A car rental agency has 30 cars to rent. The owner of the agency finds that if he charges \$40 per day, all of his cars will be rented. However, for each \$2 increase in the price, he will rent one car less. What price should he charge to maximize his revenue?

## ANSWERS

1. a) Vertex:  $(0, -5)$

$x$ -int:  $\pm\sqrt{5}$

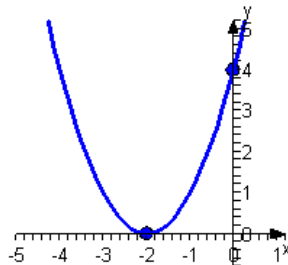
$y$ -int:  $-5$



b) Vertex:  $(-2, 0)$

$x$ -int:  $-2$

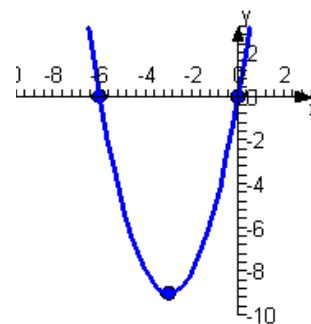
$y$ -int:  $4$



c) Vertex:  $(-3, -9)$

$x$ -int:  $0, -6$

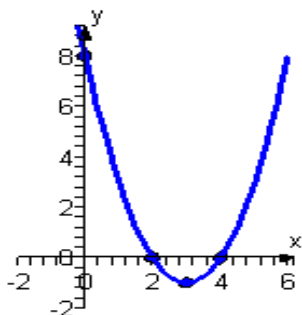
$y$ -int:  $0$



d) Vertex:  $(3, -1)$

$x$ -int:  $2, 4$

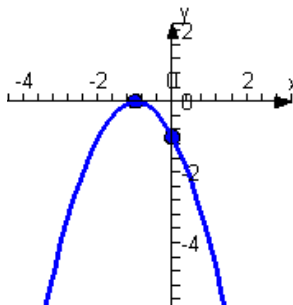
$y$ -int:  $8$



e) Vertex:  $(-1, 0)$

$x$ -int:  $-1$

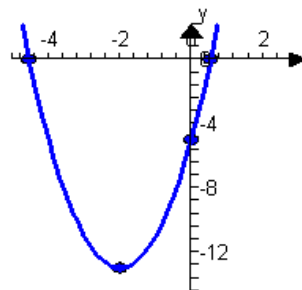
$y$ -int:  $-1$



f) Vertex:  $(-2, -13)$

$x$ -int:  $\pm\frac{1}{2}\sqrt{26} - 2$

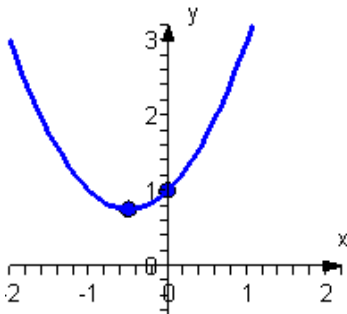
$y$ -int:  $-5$



g) Vertex:  $(\frac{-1}{2}, \frac{3}{4})$

$x$ -int: None

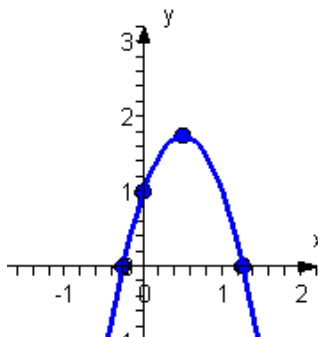
$y$ -int:  $1$



h) Vertex:  $(\frac{1}{2}, \frac{7}{4})$

$x$ -int:  $\frac{1}{2} \pm \frac{\sqrt{21}}{6}$

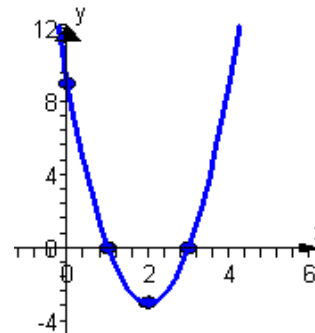
$y$ -int:  $1$



i) Vertex:  $(2, -3)$

$x$ -int:  $1, 3$

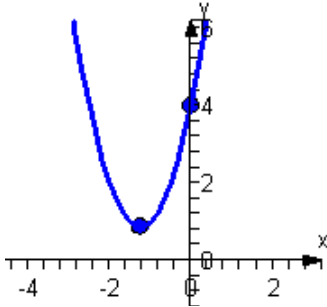
$y$ -int:  $9$



j) Vertex:  $(-\frac{5}{4}, \frac{7}{8})$

x-int: None

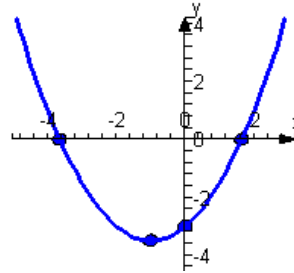
y-int: 4



k) Vertex:  $(-1, -\frac{7}{2})$

x-int:  $-1 \pm \sqrt{7}$ 

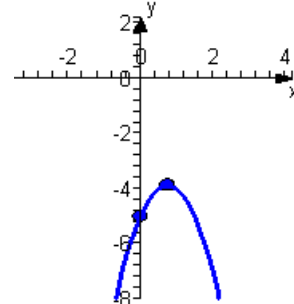
y-int: -3



l) Vertex:  $(\frac{3}{4}, -\frac{31}{8})$

x-int: None

y-int: -5



2. a) Min of  $f(-5) = -15$

d) Max of  $f(3) = -2$

3. a)  $f(x) = x^2 - 2x$

d)  $f(x) = \frac{-1}{9}x^2 - \frac{8}{9}x - \frac{25}{9}$

4. a)  $f(x) = 1 - x^2$

d)  $f(x) = 2x^2 + 4x + 4$

5. 100 and -100

6. 300m by 450m (where the fence dividing the two plots is 300 m long)

7. a)  $\frac{250}{49} \approx 5.102$  m

b)  $\frac{50}{49} \approx 1.020$  sec

8. a) 220 m

b)  $40\sqrt{11} + 40 \approx 172.7$  m

9. He should charge \$50 per day.

b) Min of  $f(15) = -4500$

e) Max of  $f(15) = 225$

b)  $f(x) = \frac{3}{2}x^2 - 6x + 5$

b)  $f(x) = x^2 + 4x + 3$

c)  $f(x) = \frac{-1}{2}x^2 + 2x - 3$

c) Max of  $f(\frac{5}{2}) = \frac{29}{4}$

f) Max of  $R(25) = 625$

c)  $f(x) = \frac{-3}{4}x^2 + 3x$

c)  $f(x) = -x^2 + 2x + 3$

d)  $f(x) = -2x^2 - 4x - 2$