MATH DEPT. SOLUTION TO TUTORIAL 12

Solution 12: Long Division

1. (i) Quotient is $2x+4$ and remainder is 0

(ii) Quotient is $4x^2 + 9x + 25$ and remainder is 105

(iii) Quotient is $x^3 + 3x^2 – 1$ and remainder is 0

(iv) Quotient is $3x + 5$ and remainder is $– 2x + 3$

(v) Quotient is $x^4 – 10x^3 – 30x^2 – 90x– 20$ and remainder is 20

(vi) Quotient is $x^2 + 2x + 4$ and remainder is $2x – 11$

2. (i) Dividing $x^3 – 4x^2 + x + 6$ by $x – 2$ gives a quotient of $x^2 – 2x – 3$ and $x^2 – 2x – 3$ factors into $(x–3)(x+1)$. Thus the other two factors are $x – 3$ and $x + 1$

(ii) Since both $x + 1$ and $x – 3$ are factors of $x^4 – 6x^3 + 9x^2 + 4x – 12$, then $(x+1)(x–3) = x^2 – 2x – 3$ is a factor. Dividing $x^4 – 6x^3+ 9x^2 + 4x – 12$ by $x^2 – 2x – 3$ gives a quotient of $x^2 – 4x + 4 = (x–2)^2$ Thus the other two factors are both $x – 2$

(iii) Dividing $x^5 + 2x^3 – 8x^2 – 16$ by $x^2 + 2$ gives a quotient of $x^3 – 8$ Now $x^3 – 8 = x^3 – 2^3 = (x–2)(x^2 + 2x + 4)$ Thus the other two factors are $x – 2$ and $x^2 + 2x + 4$

3. (i) $x – 3$ is a factor of $p(x) = x^4 + x^3 – 9x^2 – 27$ if and only if $p(3) = 0$ Now $3^4 + 3^3 – 9(3^2) – 27 = 0$. Thus $x – 3$ is a factor.

(ii) Again since $(–1)^{10} + (–1)^9 + (–1)^6 + (–1)^3 + (–1) + 1 = 0$ then the result follows.

(iii) $x – 2$ is a factor of $x^5 – 4x^4 + 5x^3 – 3x^2 + 4$ since $2^5 – 4(2)^4 + 5(2)^3 – 3(2)^2 + 4 = 32 – 64 + 40 – 12 + 4 = 0$ Now the derivative of $p(x) = x^5 – 4x^4 + 5x^3 – 3x^2 + 4$ is $p'(x) = 5x^4 – 16x^3 + 15x^2 – 6x$ and $p'(2) = 5(2)^4 – 16(2)^3 + 15(2)^2 – 6(2) = 80 – 128 + 60 – 12 = 0$. Thus $x – 2$ is also a factor of the derivative. Tne can also see this by dividing $p(x) = x^5 – 4x^4 + 5x^3 – 3x^2 + 4$ by $(x–2)^2 = x^2 – 4x + 4$. This gives $x^2 + x + 1$.

Thus $p(x) = x^5 – 4x^4 + 5x^3 – 3x^2 + 4 = (x–2)^2(x^3 + x + 1)$. The factor x-2 is said to be a factor of multiplicity 2.

Differentiating gives

$$\begin{aligned} p'(x) &= 2(x–2)(x^3 + x + 1) + (x–2)^2(3x^2+1) \\ &= (x–2)[\, 2(x^3 + x + 1) + (x–2)(3x^2+1)] \\ &= (x–2)[\, 2x^3 + 2x + 2 + 3x^3 – 6x^2+ x – 2] \\ &= (x–2)(5x^3 – 6x^2 + 2x\,) \end{aligned}$$

and it is clear that $x–2$ is a factor of $p'(x)$. In general, if a polynomial $p(x) = (x–a)^k q(x)$, then $x–a$ is said to be a factor of multiplicity k, and the result above can be generalized to read: If $(x–a)$ is a factor of multiplicity k of the polynomial $p(x)$, then $x–a$ is a factor of $p(x)$ and all its derivatives up to order k.