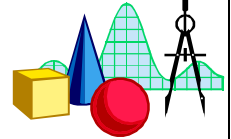




# MATH DEPT. SOLUTION TO TUTORIAL 12



## Solution 12: Long Division

1.
  - (i) Quotient is  $2x+4$  and remainder is 0
  - (ii) Quotient is  $4x^2 + 9x + 25$  and remainder is 105
  - (iii) Quotient is  $x^3 + 3x^2 - 1$  and remainder is 0
  - (iv) Quotient is  $3x + 5$  and remainder is  $-2x + 3$
  - (v) Quotient is  $x^4 - 10x^3 - 30x^2 - 90x - 20$  and remainder is 20
  - (vi) Quotient is  $x^2 + 2x + 4$  and remainder is  $2x - 11$
  
2.
  - (i) Dividing  $x^3 - 4x^2 + x + 6$  by  $x - 2$  gives a quotient of  $x^2 - 2x - 3$  and  $x^2 - 2x - 3$  factors into  $(x-3)(x+1)$ . Thus the other two factors are  $x - 3$  and  $x + 1$
  - (ii) Since both  $x + 1$  and  $x - 3$  are factors of  $x^4 - 6x^3 + 9x^2 + 4x - 12$ , then  $(x+1)(x-3) = x^2 - 2x - 3$  is a factor. Dividing  $x^4 - 6x^3 + 9x^2 + 4x - 12$  by  $x^2 - 2x - 3$  gives a quotient of  $x^2 - 4x + 4 = (x-2)^2$ . Thus the other two factors are both  $x - 2$
  - (iii) Dividing  $x^5 + 2x^3 - 8x^2 - 16$  by  $x^2 + 2$  gives a quotient of  $x^3 - 8$ . Now  $x^3 - 8 = x^3 - 2^3 = (x-2)(x^2 + 2x + 4)$ . Thus the other two factors are  $x - 2$  and  $x^2 + 2x + 4$
  
3.
  - (i)  $x - 3$  is a factor of  $p(x) = x^4 + x^3 - 9x^2 - 27$  if and only if  $p(3) = 0$ . Now  $3^4 + 3^3 - 9(3^2) - 27 = 0$ . Thus  $x - 3$  is a factor.
  - (ii) Again since  $(-1)^{10} + (-1)^9 + (-1)^6 + (-1)^3 + (-1) + 1 = 0$  then the result follows.
  - (iii)  $x - 2$  is a factor of  $x^5 - 4x^4 + 5x^3 - 3x^2 + 4$  since  $2^5 - 4(2)^4 + 5(2)^3 - 3(2)^2 + 4 = 32 - 64 + 40 - 12 + 4 = 0$ . Now the derivative of  $p(x) = x^5 - 4x^4 + 5x^3 - 3x^2 + 4$  is  $p'(x) = 5x^4 - 16x^3 + 15x^2 - 6x$  and  $p'(2) = 5(2)^4 - 16(2)^3 + 15(2)^2 - 6(2) = 80 - 128 + 60 - 12 = 0$ . Thus  $x - 2$  is also a factor of the derivative. One can also see this by dividing  $p(x) = x^5 - 4x^4 + 5x^3 - 3x^2 + 4$  by  $(x-2)^2 = x^2 - 4x + 4$ . This gives  $x^2 + x + 1$ .  
Thus  $p(x) = x^5 - 4x^4 + 5x^3 - 3x^2 + 4 = (x-2)^2(x^3 + x + 1)$ . The factor  $x-2$  is said to be a factor of multiplicity 2.  
Differentiating gives  $p'(x) = 2(x-2)(x^3 + x + 1) + (x-2)^2(3x^2 + 1)$   

$$= (x-2)[2(x^3 + x + 1) + (x-2)(3x^2 + 1)]$$

$$= (x-2)[2x^3 + 2x + 2 + 3x^3 - 6x^2 + x - 2]$$

$$= (x-2)(5x^3 - 6x^2 + 2x)$$

and it is clear that  $x-2$  is a factor of  $p'(x)$ . In general, if a polynomial  $p(x) = (x-a)^k q(x)$ , then  $x-a$  is said to be a factor of multiplicity  $k$ , and the result above can be generalized to read: If  $(x-a)$  is a factor of multiplicity  $k$  of the polynomial  $p(x)$ , then  $x-a$  is a factor of  $p(x)$  and all its derivatives up to order  $k$ .