MATH DEPT.
SOLUTION TO
TUTORIAL 11

Solution 11: Setting up Word Problems.

14. Let x be John's age.
Therefore,

$$\tfrac{1}{2}(x+2)+\tfrac{1}{3}(x-3)=20 \quad \Rightarrow \quad x = 24$$

16. Let x be the weight of the \$3.00/lb tea and let y be the weight of the \$2.75/lb tea.
Therefore,

$$x + y = 80 \quad \Rightarrow \quad y = 80 - x$$

$$\$3{\cdot}x + \$2.75{\cdot}y = \$2.90 \cdot 80 \quad \Rightarrow \quad \$3{\cdot}x + \$2.75{\cdot}(80-x) = \$232$$

$$\Rightarrow \quad \$3{\cdot}x + \$220 - \$2.75x = \$232$$

$$\Rightarrow \quad \$0.25x = \$12$$

$$\Rightarrow \quad x = 48$$

$$\Rightarrow \quad y = 80 - x = 80 - 48 = 32$$

18. Let x be the additional money invested.
Therefore,

$$8{,}000 \cdot 8\% + x \cdot 11\% = (8{,}000 + x) \cdot 9\% \quad \Rightarrow \quad 640 + \tfrac{11}{100}x = 720 + \tfrac{9}{100}x$$

$$\Rightarrow \quad \tfrac{2}{100}x = 80$$

$$\Rightarrow \quad x = \$4{,}000$$

20. Let t be the time it takes the marshal to catch up to the Daltons.
First note that when the marshal catches up with the Daltons, they have traveled the same distance.
Therefore,

$$\text{distance traveled by the marshal} = \text{distance traveled by the Daltons}$$

$$16\,t = 14 \cdot \tfrac{1}{6} + 14\,t$$

$$2\,t = \tfrac{14}{6}$$

$$t = \tfrac{7}{6} \approx 1 \text{ hour } 10 \text{ minutes}$$

22. Let x be the distance traveled upstream.
Therefore,

$$\text{time to travel upstream} + \text{time to travel downstream} = 6$$

$$\frac{x}{20+4} + \frac{x}{20-4} = 6$$

$$\frac{x}{24} + \frac{x}{16} = 6$$

$$\frac{2x+3x}{48} = 6$$

$$x = \tfrac{288}{5} = 57.6 \text{ nautical miles}$$

26. Let x be the speed of the woman's car.

First note that 50 mi/h = $\frac{220}{3}$ feet/sec.

$$6x = 6 \cdot \frac{220}{3} + 30 + 14$$
$$6x = 484$$
$$x = \frac{242}{3} \text{ feet/sec.}$$
$$x = 55 \text{ mi/h.}$$

32. Let x be the time it will take both to deliver the papers.
Therefore,

$$\frac{1}{70} + \frac{1}{80} = \frac{1}{x}$$
$$\frac{7+8}{560} = \frac{1}{x}$$
$$x = \frac{560}{15} \text{ minutes} = 37 \text{ min. } 20 \text{ sec.}$$

34. Let x be the distance of the man from the lamppost.
Therefore, using similar triangles

$$\frac{6}{5+x} = \frac{2}{5}$$
$$30 = 2(5 + x)$$
$$30 = 10 + 2x$$
$$2x = 20$$
$$x = 10$$

36. Let x be the height of the break.
Therefore, the hypotenuse of the resulting triangle is 10 - x

$$x^2 + 3^2 = (10 - x)^2$$
$$x^2 + 3^2 = 100 - 20x + x^2$$
$$20x = 100 - 9$$
$$x = \frac{91}{20} = 4.55 \text{ feet}$$

40. Let x be the increase in the radius of the circle.

$$\pi (4 + x)^2 = 10 + \pi 4^2$$
$$\pi (16 + 8x + x^2) = 10 + 16\pi$$
$$\pi x^2 + 8\pi x + 16\pi = 10 + 16\pi$$
$$\pi x^2 + 8\pi x - 10 = 0$$

This is just a quadratic equation, hence

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8\pi \pm \sqrt{64\pi^2 - 4\pi(-10)}}{2\pi} = \frac{-8\pi \pm \sqrt{64\pi^2 + 40\pi}}{2\pi}$$

However, the negative root yields a negative number for the change in the radius which is impossible, therefore the answer is $x = \frac{-8\pi + \sqrt{64\pi^2 + 40\pi}}{2\pi}$

41. Let x be the length of the side of the square piece of cardboard.
Therefore, the base of the box will be (x - 8) and note that its height will be 4.

$$\text{Volume} = 100$$
$$4 (x - 8)^2 = 100$$
$$(x - 8)^2 = 25$$
$$x^2 - 16x + 64 = 25$$
$$x^2 - 16x + 39 = 0$$
$$(x - 13)(x - 3) = 0$$

Therefore, x is either 13 or 3. However, 3 obviously cannot be an answer, so the cardboard must be 13X13 .

44. Let x be the height of the pole.
Therefore, the side of the "triangle" formed by the guy wires are x + 5.

$$x^2 + \left(\tfrac{x+5}{2}\right)^2 = (x+5)^2$$

$$x^2 - 30x - 75 = 0$$

$$15 \pm 5\sqrt{6}$$

However, the only answer for the present context is $15+5\sqrt{6}$

50. Let x be the width of the lawn.

$$(240 - 2x)(180 - 2x) = \tfrac{1}{2}(240)(180)$$

$$4(120 - x)(90 - x) = (240)(90)$$

$$(120 - x)(90 - x) = (60)(90)$$

$$10{,}800 - 210x + x^2 = 5{,}400$$

$$x^2 - 210x + 5{,}400 = 0$$

$$(x - 180)(x - 30) = 0$$

So the possible solutions are 30 or 180 feet. However, 180 feet is impossible in this context and therefore the lawn must be 30 feet wide.

54. Let x be the distance to the surface of the water.
Let t be the time it takes the stone to reach the water.

Distance falling down to the water = distance sound traveled up the well

$$16t^2 = 1090(3 - t)$$

$$16t^2 = 3270 - 1090t$$

$$16t^2 + 1090t - 3270 = 0$$

$$8t^2 + 545t - 1635 = 0$$

$$t = \frac{-545 \pm \sqrt{297{,}025 + 52{,}320}}{16} = \frac{-545 \pm \sqrt{349{,}345}}{16} \approx \begin{cases} 2.878383939 \\ -71.0033839 \end{cases}$$

However, we readily reject the negative solution and find that the distance x to the water is approximately equal to

$$16(2.878383939)^2 \approx 132.5615056 \text{ feet.}$$