CHAMPLAIN REGIONAL COLLEGE ST. LAWRENCE

# MATH DEPT. SOLUTION TO TUTORIAL 10

## Solution 10: Logarithms: graph of log function, change of base, solution of equations.

1. (i) $\log_2 16 = 4$

   (ii) $\log_5 \frac{1}{25} = -2$

   (iii) $\log_7 1 = 0$

   (iv) $\log_{10} 0.01 = -2$

   (v) $\log_{27} 9 = 2/3$

   (vi) $\log_2 \frac{1}{8} = -3$

2. (i) $\log_5 125 = 3$

   (ii) $\log_{81} 3 = 1/4$

   (iii) $\log_9 27 = 3/2$

   (iv) $\log_{10} 0.001 = -3$

3. (i) $\log_4 z - \log_4 4 = \log_4 \frac{z}{4}$

   (ii) $2 \ln 8 + 5 \ln z = \ln(8^2 z^5) = \ln(64z^5)$

   (iii) $4\ln z + 4\ln(z+5) - 2\ln(z-5) = \ln z^4 + \ln(z+5)^4 - \ln(z-5)^2 = \ln\frac{z^4(z+5)^4}{(z-5)^2}$

   (iv) $\ln(x-2) - \ln(x+2) = \ln \frac{x-2}{x+2}$

4. (i) $\log 10\ 6 = \log_{10} 2(3) = \log_{10} 2 + \log_{10} 3 = 0.78$

   (ii) $\log_{10} 30 = \log_{10} (2)(3)(5) = \log_{10} 2 + \log_{10} 3 + \log_{10} 5 = 1.48$

   (iii) $\log_{10} (0.6) = \log_{10} \frac{3}{5} = \log_{10} 3 - \log_{10} 5 = -0.18$

   (iv) $\log_{10} 8 = \log_{10} 2^3 = 3 \log_{10} 2 = 3(0.30) = 0.90$

   (v) $\log_{10} \frac{1}{27} = \log_{10} 1 - \log_{10} 27 = 0 - \log_{10} 3^3 = -3 \log_{10} 3 = -1.44$

5. (i) $\log_4 12 = \frac{\ln 12}{\ln 4} = 1.79$

   (ii) $\log_6 317 = \frac{\ln 317}{\ln 6} = 3.21$

   (iii) $\log_2 19 = \frac{\ln 19}{\ln 2} = 4.25$

6. (i) $x = 4^3 = 256$

   (ii) $5x = 5^2$ or $x = 5$

(iii) $\log_2 \frac{x+8}{x^2+1} = 4$ or $\frac{x+8}{x^2+1} = 2$ or $x+8 = 2x^2 + 2$ ; $2x^2 - x - 6 = 0 \Rightarrow (2x+3)(x-2) = 0$ , $x = -3/2$ or $x = 2$

(iv) $\ln \frac{x+3}{x(x-1)} = 0$ or $\frac{x+3}{x(x-1)} = 1$ ; $x+3 = x^2 - x$ ; $x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0$ ; $x = 3$ or $x = -1$ . But $x = -1$ is not a solution since $\ln x$ is defined only for positive values of x.

(v) $\log_4 \frac{x}{x-1} = 1/2$ or $\frac{x}{x-1} = 4^{1/2} = 2$ . Then $x = 2(x-1)$ or $x = 2$

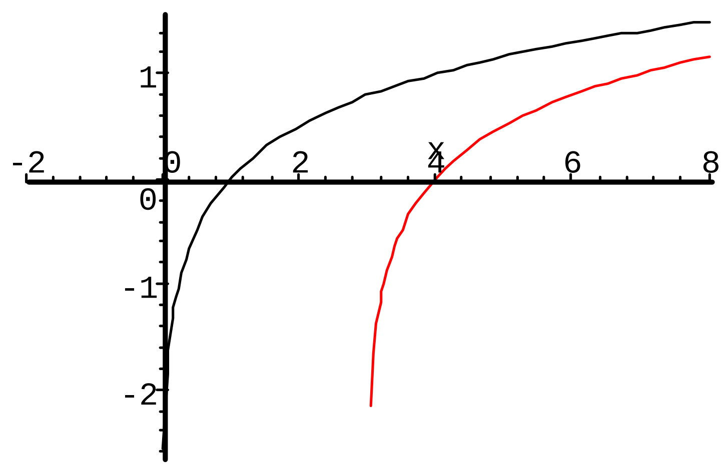
7. (i) $e^x = 10$ or $x = \ln 10 = 2.303$

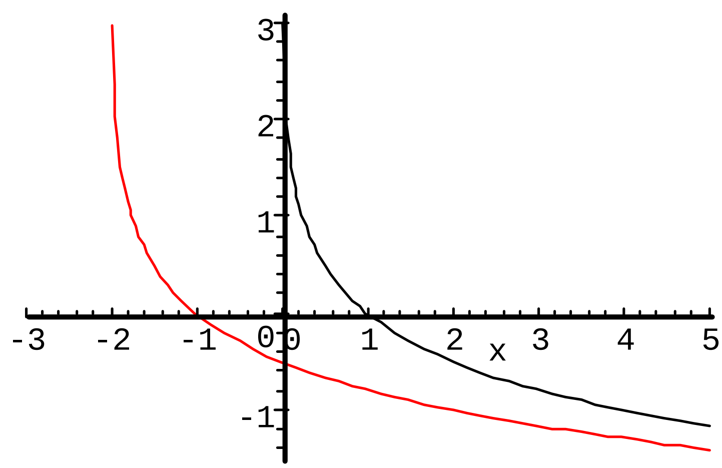
(ii) $e^{2x} = 15$ ; $2x = \ln 15$, $x = \frac{\ln 15}{2} = 1.354$

(iii) $3^{2x} = 80$ or $2x(\ln 3) = \ln 80$ , $x = \frac{\ln 80}{2(\ln 3)} = 1.994$

(iv) $4^{3x-5} = 2^{2x}$ or $(3x-5)\ln 4 = 2x (\ln 2)$ or $2(3x-5)(\ln 2) = 2x(\ln 2) \Rightarrow [2(3x-5) - 2x] \ln 2 = 0$
$6x - 10 - 2x = 0$ , $x = 5/2$ or $(2^2)^{3x-5} = 2^{2x}$ . Then $2^{2(3x-5)} = 2^{2x}$ and $2(3x-5) = 2x$. $x = 5/2$

8.



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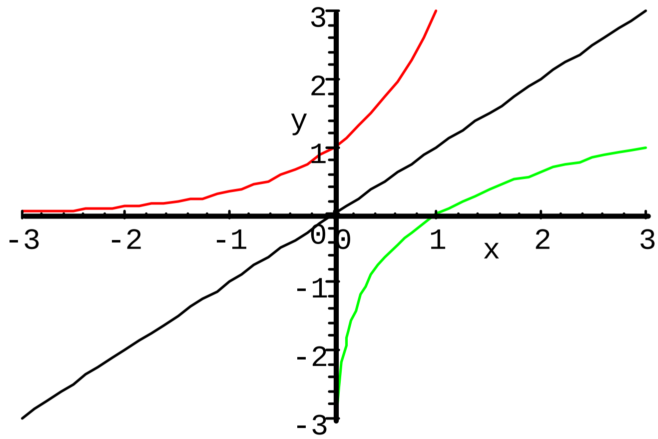


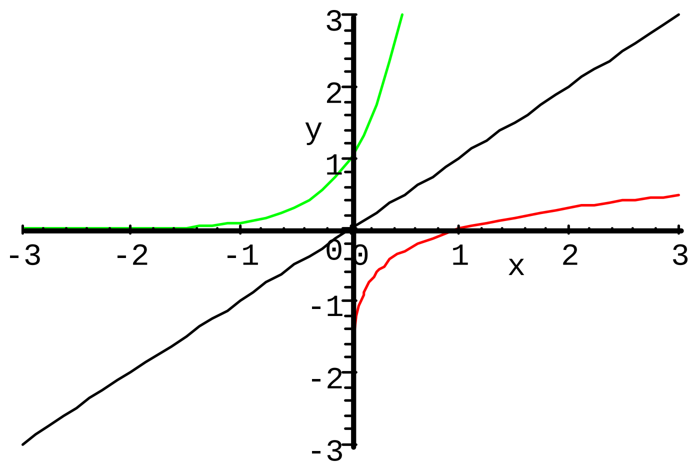


10. Since $P = 105{,}300\ e^{0.015t}$ then replace P by 150,000 and solve for t

$150{,}000 = 105{,}300\ e^{0.015t}$

$e^{0.015t}\ \frac{150000}{105300}$ and $0.015t = \ln\left(\frac{150000}{105300}\right)$

$t = \frac{\ln\left(\frac{150000}{105300}\right)}{0.015} = 23.6$ years ( approximately )

11. Since $N = 250\ (2^{kt})$ then we must first find the value of k

We are given that $N = 330$ when $t = 10$. This gives

$330 = 250(2^{10k})$ . Taking logarithms we have

$\ln 330 = \ln 250 + 10k\ (\ln 2)$ or $k = \frac{\ln 330 - \ln 250}{10 \ln 2}$

$k = 0.04$ ( approximately )

Thus $N = 250(2^{0.04t})$

When the population is 500 we have

$500 = 250(2^{0.04t})$ or $2^{0.04t} = 2$

Thus $0.04t = 1$ and $t = \frac{1}{0.04} = 25$ hours .