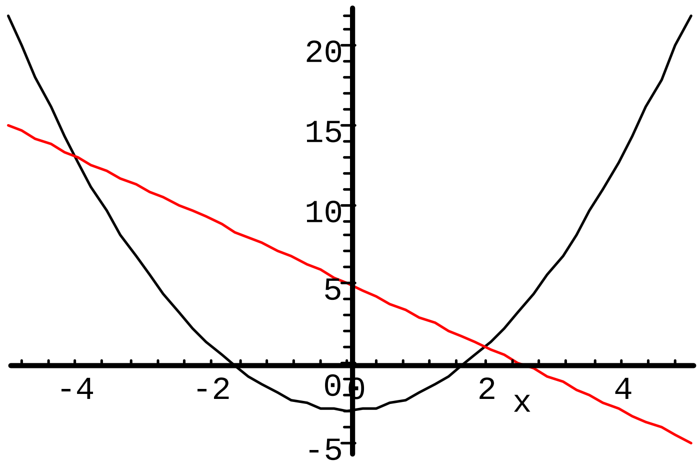
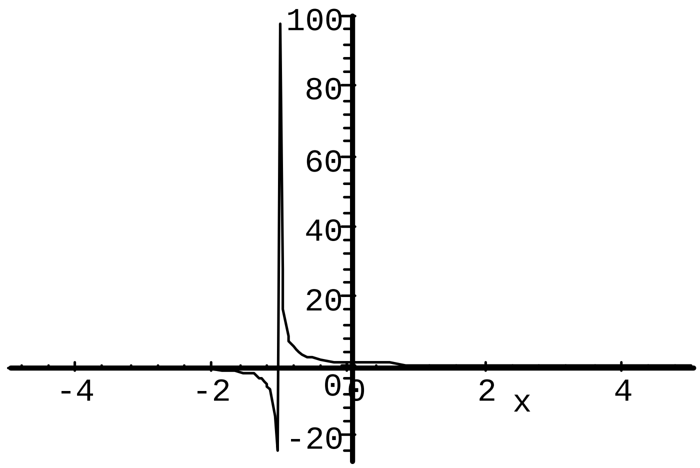
CHAMPLAIN REGIONAL COLLEGE ST. LAWRENCE

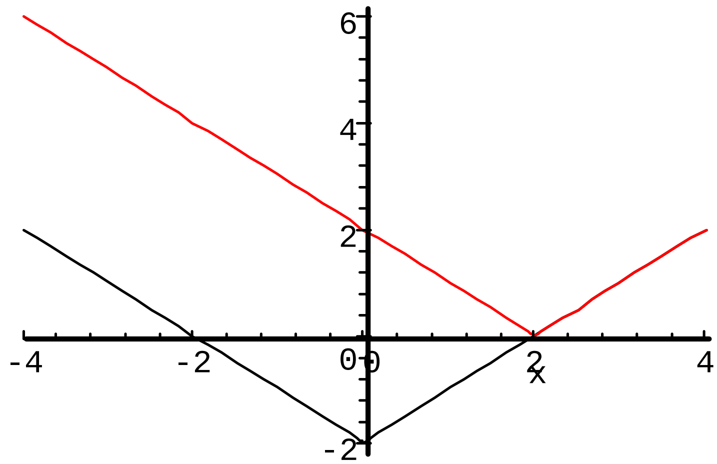
MATH DEPT. SOLUTION TO TUTORIAL 6

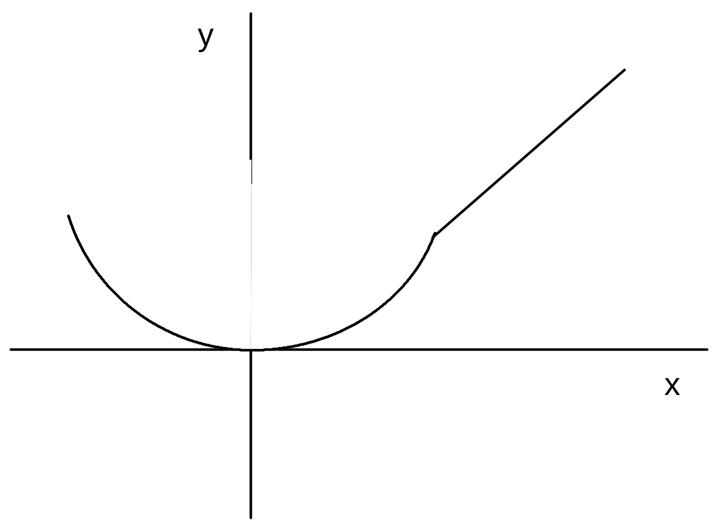
Solution 6: Functions: graphs, addition, product, division, composite functions, inverse functions.

1.









2. (i) Domain R (all reals), Range $y \geq -5/4$
(ii) Domain $x \geq -2$, Range $y \geq 0$
(iii) Domain $x \neq 2$, Range $R - \{2\}$
(iv) Domain $|x| \geq 2$ Range $y \geq 0$
(v) Domain $x \neq 0, x \neq 5$, Range $y \neq 0, y \neq -1$

3. (i) $f+g = (x-3) + (2x+1) = 3x-2$
$f-g = (x-3) - (2x+1) = -x-4$
$fg = (x-3)(2x+1) = 2x^2 - 5x - 3$
$f/g = (x-3)/(2x+1)$
Domain is R (the intersection of the domains for f and g) except for f/g where the domain is R - {-1/2} i.e. the intersection of the domains of f and g excluding those values in the domain of g where g = 0 .

(ii) $f+g = (x^2-1) + (x^2+4x+3) = 2x^2 + 4x + 2$; $f-g = -4x-4$
$fg = (x^2-1)(x^2+4x+3) = x^4 + 4x^3 + 2x^2 - 4x - 3$
$f/g = (x^2-1)/(x^2+4x+3)$
The domain is R for each function except for f/g where the domain is $R - \{-3, -1\}$

(iii) $f+g = \sqrt{x^2-4} + \sqrt{9-x^2}$
$f-g = \sqrt{x^2-4} - \sqrt{9-x^2}$
$fg = \sqrt{x^2-4}\sqrt{9-x^2}$

$f/g = \dfrac{\sqrt{x^2-4}}{\sqrt{9-x^2}}$

The domain for the first three is the intersection of $x \le -2$, $x \ge 2$ ($x^2 - 4 \;\; 0$) and $-3 \;\; \le x \le 3$ ($9 - x^2 \ge 0$)
i.e. $-3 \le x \le -2 \approx 2 \le x \le 3$
The domain of f/g is the same as that of the others except for the values of x for which g = 0 i.e. $x \ne \pm 3$
Thus the domain of f/g is $-3 < x \le -2 \approx 2 \le x < 3$

4. (i) $f(g(x)) = 2x + 1 - 3 = 2x - 2$, $g(f(x)) = 2(x-3) + 1 = 2x - 5$
Domain is R in both cases

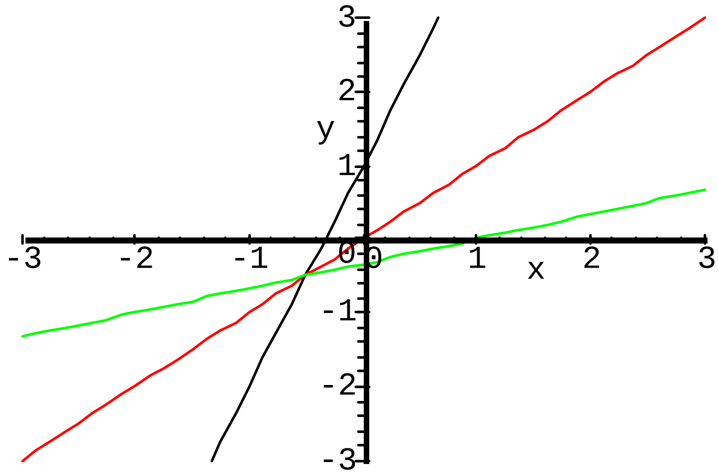
(ii) $f(g(x)) = \sqrt{|\,x\,|}$, $g(f(x)) = |\sqrt{x}\,| = \sqrt{x}$
The domain of f(g(x)) is R while the domain of g(fx)) is $x \ge 0$

(iii) $f(g(x)) = (3x^2 - 2x + 1)^5$, $g(f(x)) = 3(x^5)^2 - 2(x^5) + 1 = 3x^{10} - 2x^5 + 1$
The domain in both cases is R.

(iv) $f(g(x)) = \sqrt{x^2 - 4 - 5} = \sqrt{x^2 - 9}$, $g(f(x)) = (\sqrt{x-5}\,)^2 - 4 = x - 9$
The domain of f(g(x)) is $|\,x\,| \ge 3$ while the domain of g(f(x)) is all $x \ge 5$

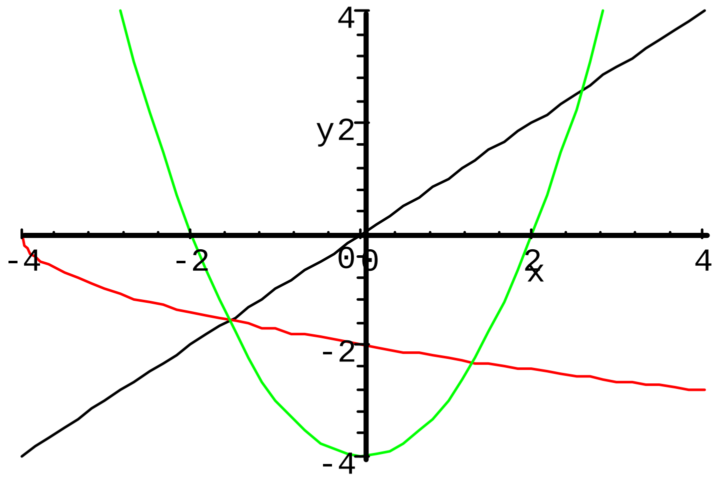
(v) $f(g(x)) = (x^2 - 1)^{1/3}$, $g(f(x)) = (x^{1/3})^2 - 1 = x^{2/3} - 1$
The domain of both functions is R .

5. (i) If $y = 3x + 1$ then we set $x = 3y + 1$ and solve to obtain $y = \dfrac{x-1}{3} = f^{-1}(x)$



Domain and Range for f and f^{-1} is R .

(ii) Note that the domain of f is given as $x \le 0$. Then the range is $y \ge -4$.
The inverse f^{-1} will have domain all $x \ge -4$ and range all $y \le 0$.
Now interchanging the x and y variables and solving we have $x = y^2 - 4$ or $y^2 = x + 4$ or $y = \sqrt{x+4}$
(remember y 0)



(iii) The domain is given as $x \geq 1/2$ (f is not defined for $x < 1/2$) Then the range will be $y \geq 0$. Thus the inverse function will have domain $x \geq 0$ and range $y \geq 1/2$

The inverse function is found by interchanging x and y and solving for y. Thus $x = \sqrt{2y - 1}$ or $2y - 1 = x2$; $y = \frac{(x2 + 1)}{2}$