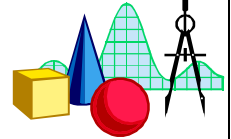


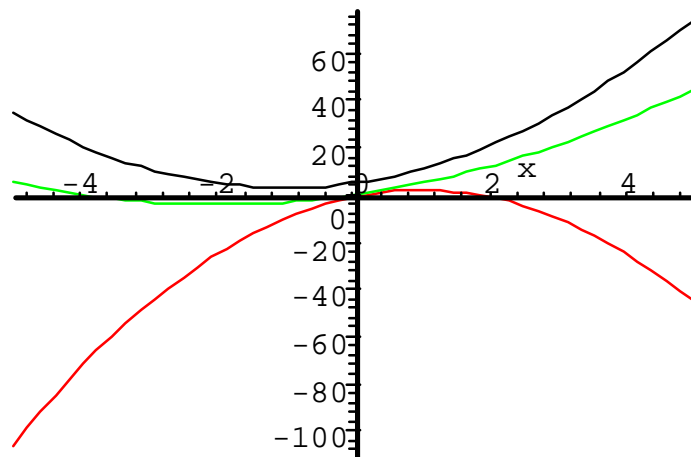


MATH DEPT. SOLUTION TO TUTORIAL 4



Solution 4: Linear and quadratic equalities and inequalities, completing the square, quadratic formula.

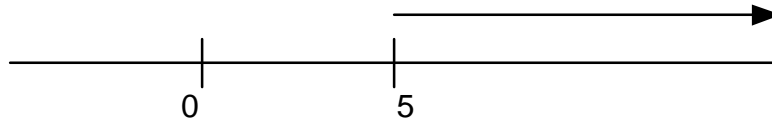
1.



2. (i) $3 + 2n = 5n + 7$ or $3n = -4$; $n = -4/3$
(ii) $2x + 1 = 3x + 5$ or $x = -4$
(iii) $x - (2x + 1) = 3x - 10$ or $x - 2x - 1 = 3x - 10$; $-x - 1 = 3x - 10$, $9 = 4x$ or $x = 9/4$
(iv) $\frac{6t + 1}{4t - 1} = \frac{3t + 8}{2t - 4}$; multiplying both sides by $(4t - 1)(2t - 4)$ we have $(6t + 1)(2t - 4) = (3t + 8)(4t - 1)$ or $12t^2 - 22t - 4 = 12t^2 + 29t - 8$ or $-22t - 4 = 29t - 8$; $51t = 4$ or $t = 4/51$
3. (i) $x^2 - 4x + 2 = x^2 - 4x + 4 - 2 = (x - 2)^2 - 2$, Thus $x^2 - 4x + 2 = 0$ implies that $(x - 2)^2 - 2 = 0$ or $(x - 2)^2 = 2$; $(x - 2) = \pm\sqrt{2}$ and $x = 2 \pm\sqrt{2}$
(ii) $2x^2 + 5x + 3 = 2(x^2 + \frac{5}{2}x) + 3 = 2(x^2 + \frac{5}{2}x + \frac{25}{16}) + 3 - \frac{25}{8} = 2(x + \frac{5}{4})^2 - \frac{1}{8}$ Then $2x^2 + 5x + 3 = 0$ implies that $2(x + \frac{5}{4})^2 - \frac{1}{8} = 0$ or $2(x + \frac{5}{4})^2 = \frac{1}{8}$, $(x + \frac{5}{4})^2 = \frac{1}{16}$, $x + \frac{5}{4} = \pm\sqrt{\frac{1}{16}}$ or $x = -\frac{5}{4} \pm \frac{1}{4}$ i.e. $x = -\frac{3}{2}$, $x = -1$
(iii) $3x^2 - 2x - 2 = 3(x^2 - \frac{2}{3}x) - 2 = 3(x^2 - \frac{2}{3}x + \frac{1}{9}) - 2 - \frac{1}{3} = 3(x - \frac{1}{3})^2 - \frac{7}{3}$, Then $3x^2 - 2x - 2 = 0$ implies that $3(x - \frac{1}{3})^2 - \frac{7}{3} = 0$ or $3(x - \frac{1}{3})^2 = \frac{7}{3}$; $(x - \frac{1}{3})^2 = \frac{7}{9}$, $x - \frac{1}{3} = \pm\sqrt{\frac{7}{9}}$ or $x = \frac{1}{3} \pm\sqrt{\frac{7}{9}}$; $x = \frac{1 \pm\sqrt{7}}{3}$

4. (i) $x^2 + 6x + 1 = 0$; $x = \frac{-6 \pm \sqrt{36 - 4(1)(1)}}{2}$ or $x = -3 \pm 2\sqrt{2}$
(ii) $4t^2 + t + 1 = 0$; $t = \frac{-1 \pm \sqrt{1 - 4(4)(1)}}{8} = \frac{-1 \pm \sqrt{-15}}{8}$ complex solutions.
(iii) $3x^2 - 2x - 2 = 0$; $x = \frac{2 \pm \sqrt{4 - 4(3)(-2)}}{6} = \frac{2 \pm \sqrt{28}}{6} = \frac{1 \pm \sqrt{7}}{3}$

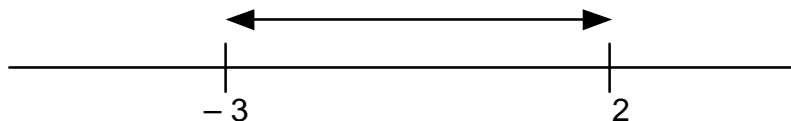
5. (i) $2x - 2 \geq 3 + x$ implies that $2x - x \geq 3 + 2$ or $x \geq 5$



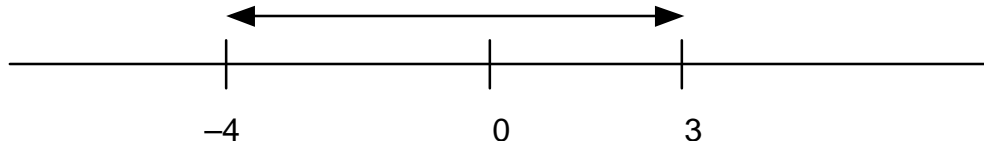
- (ii) $2x - 3 \leq x - 1$ and $2x + 2 \geq x - 1$ is two inequalities

$$\begin{array}{l} 2x - 3 \leq x - 1 \\ 2x - x \leq -1 + 3 \\ x \leq 2 \end{array} \quad \text{and} \quad \begin{array}{l} x - 1 \leq 2x + 2 \\ x - 1 - 2 \leq 2x - x \\ -3 \leq x \end{array}$$

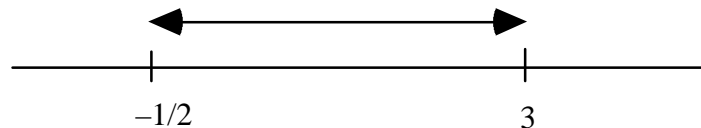
Combining the two inequalities (remember that the solution set for "and" is the intersection of the two solution sets) we have the solution set as $-3 \leq x \leq 2$



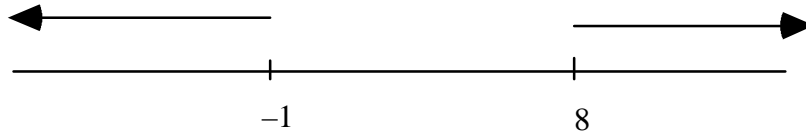
- (iii) $x^2 + x - 12 < 0$ or $(x + 4)(x - 3) < 0$ Then either $x + 4 > 0$ and $x - 3 < 0$ or $x + 4 < 0$ and $x - 3 > 0$.
If $x + 4 > 0$ and $x - 3 < 0$ then $x > -4$ and $x < 3$. Thus the solution set is $-4 < x < 3$.
If $x + 4 < 0$ and $x - 3 > 0$ then $x < -4$ and $x > 3$. Thus the solution is \emptyset .
Therefore our solution set is $-4 < x < 3$



- (iv) $2x^2 - 5x + 3 < 0$ or $2x^2 - 5x - 3 < 0$; $(2x + 1)(x - 3) < 0$.
Then either $(2x + 1) < 0$ and $(x - 3) > 0$ or $(2x + 1) > 0$ and $(x - 3) < 0$.
If $2x + 1 < 0$ and $x - 3 > 0$, then $x < -\frac{1}{2}$ and $x > 3$. Thus the solution set is $-\frac{1}{2} < x < 3$.
If $2x + 1 > 0$ and $x - 3 < 0$ then $x > -\frac{1}{2}$ and $x < 3$, again the \emptyset .
Thus our solution set is $-\frac{1}{2} < x < 3$.



- (v) $x(x - 7) > 8$ or $x^2 - 7x - 8 > 0$. Then $(x - 8)(x + 1) > 0$. Then either $(x - 8) > 0$ and $(x + 1) > 0$ or $x - 8 < 0$ and $x + 1 < 0$.
If $x - 8 > 0$ and $x + 1 > 0$ then $x > 8$ and $x > -1$. If $x > 8$ then it is certainly > -1 , Thus the solution set to the first set of inequalities is $x > 8$.
Now, if $x - 8 < 0$ and $x + 1 < 0$ then $x < 8$ and $x < -1$.
Now if $x < -1$ it is certainly < 8 . thus the solution set to the second set of inequalities is $x < -1$ We thus arrive at the solution set as $x < -1$ or $x > 8$.



(vi) $\frac{1}{x+2} > \frac{3}{x+1}$

There are a number of cases.

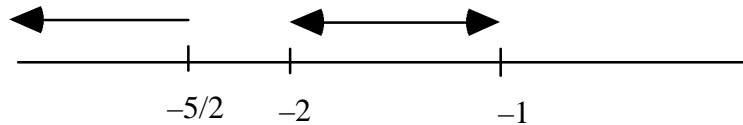
Case I $x + 2 > 0$ and $x + 1 > 0$ i.e. $x > -1$ Then we can multiply both sides by $(x+1)(x+2)$ without changing the inequality sign. We obtain $x+1 > 3(x+2)$ or $x+1 > 3x+6$; $2x < -5$ or $x < -\frac{5}{2}$. Thus we have an empty solution set since x must be > -1 .

Case II $x + 2 > 0$ and $x + 1 < 0$. Then $x > -2$ and $x < -1$ or $-2 < x < -1$.Then multiplying both sides by $x + 2$ leaves the inequality sign the same while multiplying both sides by $x + 1$ reverses the inequality sign. ($(x+2)(x+1) < 0$) Then $3(x+2) > x+1$ or $3x+6 > x+1$; $2x > -5$ and $x > -\frac{5}{2}$. Combining $x > -\frac{5}{2}$ with $-2 < x < -1$ we obtain $-2 < x < -1$

Case III $x + 2 < 0$ and $x + 1 > 0$. Then $x < -2$ and $x > -1$. No solution.

Case IV $x + 2 < 0$ and $x + 1 < 0$. Then $x < -2$ and $x < -1$. i.e. $x < -2$. (if $x < -2$ then it is < -1) . Then $(x+2)(x+1) > 0$ and multiplying both sides by $(x+2)(x+1)$ leaves the inequality sign unchanged. Then $x+1 > 3(x+2)$ or $x < -\frac{5}{2}$ as in Case I . Thus our solution set is $x < -\frac{5}{2}$ (since if $x < -\frac{5}{2}$ then it is < -2) .

Combining these cases we have as a solution set $x < -\frac{5}{2}$ or $-2 < x < -1$



REMARK: One can solve inequalities such as $x^2 + x - 12 > 0$ by first graphing the function $y = x^2 + x - 12$. Then from the graph the solution can be easily observed as those values of x for which the graph lies above the x axis.